

Lab work in photonics

Imaging systems 1

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- DSPI -
Digital Speckle Pattern
Interferometry

Answering the questions didn't replace a redaction work for the compte-rendu. These questions are essentially there to guide you in the labwork and for that redaction.

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INTRODUCTION

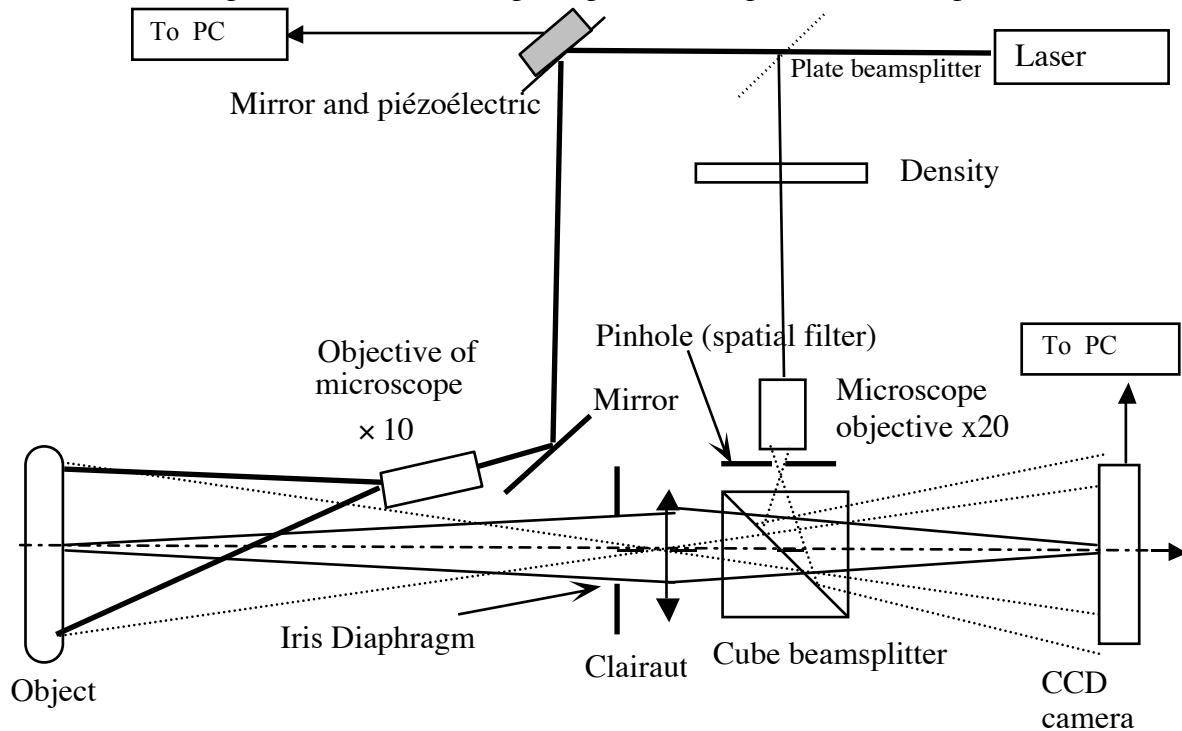
The goal of this Practical Labwork is to build an experimental device capable of emphasizing and measuring sub-micronic bending of a scattering object. The principle of this non-contact and real time measurement technique is based on the analysis of the phase of a speckle grain by an interferometrical method using a laser source. This Practical Labwork will also be the occasion to revisit or discover some general notions concerning the speckle phenomenon.

I. Experimental set-up

In its principle an interferometric optical set-up allows the measurement of the deformations of an object thanks to the analysis of the interference fringes and of their evolution when the object is deformed. However, this type of direct analysis requires to resolve the interference fringes, what is possible only with regular object and especially objects with a particularly smooth surface state. As soon as we will work with a rough and scattering object, the wavefront coming from the object will be distorted and the interference pattern will be covered by a random interference structure coming from the coherent scattering of light on the object: **the Speckle**. The fringes mixed with this random phenomenon cannot be distinguished anymore and thus cannot be analysed directly to extract the information on the object deformation. Nevertheless, the interferometric information still exists and can be processed to extract the desired information, thanks to the technique of "**Digital Speckle Pattern Interferometry**" (DSPI) that will be studied during this Labwork.

1. Scheme of the set-up

The following sketch describes the principle of the experimental set-up.



On that scheme, we can distinguish the two following optical paths:

- the first one, located in the extension of the laser ($\lambda=633\text{nm}$), is used in order to light up the object and to form its image on the retina of the CCD camera,
- the second one is used so as to form the reference wave.

We find here the principle of classical holography (on photographic plate). The reference wave and the wave coming from the object interfere in the plane of the CCD. The reference wave enables us to code the phase of the speckle grains found in the object's image. The light source used is a laser so as to get a satisfactory coherence length and a high illumination of the object. An acquisition card enables us to take in the picture of the interferences field, which results from it, before and after the object's bending.

In the CCD camera plane, the intensity related to the interference pattern writes as:

$$I_{Inter}(x, y, t) = I_0(x, y) \left[1 + m(x, y) \cos \left((\vec{k}_R - \vec{k}_S) \cdot \vec{r} + \varphi_{Obj}(x, y) + \Delta\varphi(x, y, t) \right) \right]$$

Where $(\vec{k}_R - \vec{k}_S) \cdot \vec{r}$ (with $\vec{r} = (x, y)$) corresponds to the pattern of interference fringe between the two beams that propagates in the average direction \vec{k}_R for the reference and \vec{k}_S for the signal. That interference pattern is modulated by the speckle structure characterized by $\varphi_{Obj}(x, y)$ the random phase corresponding to the light scattering on the rough object. Finally, $\Delta\varphi(x, y, t)$ (supposed to be a slowly varying function of (x, y)) is **the phase change due to deformation that we want to measure**. The interference pattern is at least characterized by the mean intensity of the beams $I_0(x, y)$ and the modulation $m(x, y)$ of the fringes, which is supposed to be not modified by the deformation of the object.

During the measurement, that interference signal is detected by the CCD Camera constituted of pixels of size P_e . Each pixel integrates the interference signal incident on its surface S_p , what imposes a condition on the size of speckle grains and the fringe spacing in order to access the deformation signal we are interested in. Indeed, the detected signal by a pixel of coordinates (x_p, y_p) is given by:

$$I_{Pixel}(x_p, y_p, t) = \int_{S_p} I_0(x, y) \left[1 + m(x, y) \cos \left((\vec{k}_R - \vec{k}_S) \cdot \vec{r} + \varphi_{Obj}(x, y) + \Delta\varphi(x, y, t) \right) \right] dx dy$$

To access to $\Delta\varphi(x, y, t)$, we absolutely need that the terms $(\vec{k}_R - \vec{k}_S) \cdot \vec{r}$ and $\varphi_{Obj}(x, y)$ vary slowly on the surface of the pixel, so that the term depending on the cosinus of the phase doesn't average to zero. That then imposes first, that **fringe spacing has to be greater than the size of the pixels of the camera**, second that the **size of the speckle grains is also greater than the size of the same pixels of the camera**.

Under these conditions the intensity detected by each pixel of the camera is given by:

$$I_{Pixel}(x_p, y_p, t) = I_0(x_p, y_p) \left[1 + m(x_p, y_p) \cos \left(\varphi_0(x_p, y_p) + \Delta\varphi(x_p, y_p, t) \right) \right]$$

where $\varphi_0(x_p, y_p)$ is a stationary phase function that can randomly vary from one pixel of the camera to the other.

2. Alignment of the object path

The object is a piezoelectric ceramic which we want to study the deformation induced by an applied voltage.

- ➔ Block the reference path.
- ➔ Construct the object illumination path.

The illumination beam will be aligned such that the scattering lobe is correctly directed towards the objective diaphragm. This diaphragm is an iris diaphragm placed aside the imaging Clairaut of focal length **100mm**. The diameter of this diaphragm is thus adjustable and a graduation indicating its size can be found on its side.

- ➔ Precisely look for the best imaging by translating the CCD Camera.

Q1- How can we perform a precise imaging? What is the influence of the diameter of the diaphragm on the precision of the CCD positioning to obtain the best image?

3. Alignment of the reference path and observation of the fringe pattern

The reference beam must superimpose with signal beam issuing from the object on the camera covering it uniformly.

→ Block the signal path.

→ realize the alignment of the laser purification in order to have a powerful reference beam covering largely the CCD camera and verify the position of the beamsplitter so that the beam is superimposed with the object beam arriving on the camera.

→ Place and align the neutral densities in order to adjust the intensity equilibrium between the two set-up paths.

The interferometric set-up is of the Mach-Zehnder family, the fringe spacing then depends on the mean angle between the interfering beams. Nevertheless, taking into account its practical structure, one can also describe the set-up as a "Young hole" type interferometric set-up.

Q2- What are the two elements of the set-up that can materialize the "Young holes"? How can the fringe spacing be varied experimentally?

→ Close the diaphragm at smallest size in order to observe large diameter speckle grains and observe the image of the speckle pattern **in presence of the reference beam**.

→ Observe the shape of the fringes in the speckle pattern when the transversal position (vertically and horizontally) of the diaphragm is varied.

Each speckle grain can be locally assimilated to a plane wave having an almost constant phase (the coherence volume) that randomly varies from one speckle grain to the other.

Q3- How can you show and experimentally verify that the speckle grain phase is almost uniform.

II. Study of the properties of the speckle pattern

*In this part, **the reference beam is cut, and we work with signal beam issued from the target only. All the measurement of Section II will be performed for 4 different sizes of the diaphragm : 1, 3, 5 et 10 mm.***

The set-up allows a detailed study of the speckle pattern observed by the CCD retina. That study will lead to the best choice of the speckle pattern parameters in order to obtain the best results during speckle interferometry measurements.

Q4- The mean size of the speckle grain is given by the size of the diffraction spot.

Calculate the theoretical mean speckle diameter as a function of the geometrical and optical parameters of the used optical set-up.

1. Mean diameter of the speckle grains

We will study precisely the CCD camera illumination pattern with *Matlab*. The software allows to directly collect the image digitized by a video acquisition card.

→ Launch *Matlab* to acquire and process the speckle pattern images.

→ Measure approximately the mean diameter of the speckle grains in the image as a unit of pixels.

Some potentially useful Matlab commands:

image_live	<i>Display the image in real time in Matlab (first command to launch)</i>
img=videosnap(vid);	<i>Acquire and display a video image in Black and White (with zoom and grid)</i>
img=videosnap2(vid);	<i>Acquire a video image without displaying it</i>
affimage(img);	<i>Display the image "img"</i>
centre_img=imcrop;	<i>Capture a chosen zone of the image</i>

Q5- Deduce from that measurement an approximate value of the mean diameter of the speckle grains (the sampling step P_e , is fixed by the CCD camera pixel size, i.e. $P_e = 8.3\mu\text{m}$). What is the main reason for the uncertainty of the measurement?

The mean size of the speckle grains can also be obtained with a higher precision, by a measurement of the width of the autocorrelation of the illumination pattern. That function can be calculated easily using the Fourier Transform properties:

$$Cor(E(\vec{r}_1), E(\vec{r}_2)) = TF^{-1}[TF[E(\vec{r})].TF[E(\vec{r})]^*]$$

→ Briefly validate that method.

→ Realize the calculation of that function with Matlab for the central part of the image (take the largest part where the illumination can be considered uniform).

<code>Tfimg=fftshift(fft2(fftshift(centre_img))); autocorr=abs(fftshift(iff2(Tfimg.*conj(Tfimg))));</code>	<i>Autocorrelation of the image (detailed command)</i>
<code>auto=autocorrelation(vid);</code>	<i>Autocorrelation of the image (direct command)</i>
<code>improfile;</code>	<i>Display a profile of the previous image</i>

Q6- Compare the speckle grain size obtained with the width of the autocorrelation function to the one obtained previously and to the theoretical value. Comment.

2. Study of the sampling of the speckle pattern by the CCD camera retina.

Another way to evaluate the speckle grain size consists in going to the spatial frequency domain by calculating the 2D Fourier Transform (or equivalently the inverse Fourier Transform) of the illumination pattern.

Q7- What is the link between the amplitude in the CCD camera plane $A_C(x,y)$ and the spatial repartition of amplitude $A_{pup}(X,Y)$ of the optical wave in the diaphragm plane, deduce from it the value of the intensity $I_C(x,y)$.

Q8- Which quantity can be related to the Inverse Fourier Transform of $I_C(x,y)$? What spatial shape do you expect for it?

→ Realize the Fourier Transform of the centre of the image of the speckle pattern for the different studied iris diaphragm diameters.

<code>fft_img=fft_dsipi(vid);</code>	<i>Realize the Fourier Transform of an image</i>
<code>profil_moyen;</code>	<i>Realize the mean profile of the previous image</i>

Q9- Explain why the expected width of this Fourier Transform is determined by the cut-off frequency of the ideal modulation transfer function of the objective (i.e. $1/\lambda N_{image}$ where N_{image} is the image aperture number). Verify that result for several diaphragm diameters.

The illumination pattern in the plane of the camera is sampled by the camera pixels, what induces an additional cut-off frequency, that competes with the cut-off frequency related to the set-up diaphragm.

Q10- Briefly explain why a spatial frequency spectrum pixel corresponds to: $\frac{F_e}{N_e} = \frac{1}{N_e P_e}$

with $P_e = 8,3\mu m$ the pixel size and N_e the number of points used to calculate the FFT.

Q11- What is the maximum spatial frequency measured by the CCD camera? What does it correspond to in practice when we calculate the Fourier Transform of the speckle pattern?

Q12- In the following we will try to measure the spatial deformation of the scattering target and we have seen that the speckle grain size has to be taken larger than pixel size. What criteria can we use on the FFT image to make that choice. What diaphragm diameter should we take?

III. Study of the properties of the interference pattern

In this part, the reference beam is put on again, we now work with the two beams.

The CCD camera, as any optical sensor, is only sensitive to the intensity of the illumination. Nevertheless, adding the reference beam allows to access to the phase of the speckle pattern. The interferometric set-up is here of the Mach-Zehnder type, the fringe spacing

then depends on the mean angle between the two interfering beams.

Q13- What condition has the fringe spacing to verify compared to the CCD camera pixel size to verify in order for them to be observable? What is the maximum angle between the beams allowing to verify that condition?

→ Place yourselves in a situation where fringe spacing is small. Realize and compare the Fourier Transform of images "speckle + reference", "speckle alone" et "reference alone".

Q14- Write the intensity in the plane of the camera $I_C(x,y)$ as a function of the amplitude spatial pattern $A_{Pup}(X,Y)$ of the optical wave in the plane of the diaphragm and of the amplitude of the reference wave $A_{Ref}(x,y)$ (that will be supposed uniform in the plane of the camera, $A_{Ref}(x,y)=A_{Ref0}$).

Q15- Calculate the Inverse Fourier Transform of $I_C(x,y)$. Identify the different terms in that expression with the observed structures and determine which structures correspond to the fringe pattern. What parameter governs the distance between those structures?

Q16- Why is the central structure two times larger? Explain that result using the calculation of the Inverse Fourier Transform of $I_C(x,y)$ in presence of the reference wave.

Q17- Propose a method to adjust the position of the diaphragm and its diameter.

IV. Speckle interferometry – Study of the deformation of an object

The set-up is now supposed optimally aligned; (i) diameter of the speckle grains of the order of the camera pixel size and (ii) fringe spacing larger than the speckle grain size. We will thus be able to use the set-up to measure the deformation or the vibration of the target object. Each pixel ($x=x_p, y=y_p$) of the camera receive an intensity:

$$I_{CCD}(x, y, t) = I_0(x, y) [1 + m(x, y) \cos(\varphi_0(x, y) + \Delta\varphi(x, y, t))]$$

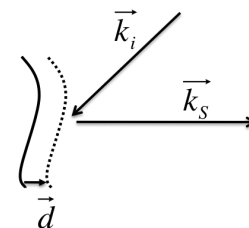
with $\varphi_0(x, y)$ the random phase corresponding to light scattering on the rough object modulated by the interference term, and $\Delta\varphi(x, y, t)$ is the phase change induced by the deformation of the object and supposed to be a slowly varying (x, y) function.

In the case of a deformation measurement, we measure the variation of the object shape between two instants separated by a time Δt , comparing $I_{CCD}(x, y, \Delta t)$ with $I_{CCD}(x, y, 0)$ (supposing that $\Delta\varphi(x, y, 0) = 0$).

1. Sensitivity of the phase to local deformation

Let's consider:

- a rough plane surface (perpendicular to the z axis),
- an incident plane wave with wave vector \vec{k}_i tilted compared to the z axis,
- a scattered plane wave with a wave vector \vec{k}_s .



Q18- Show that the variation of the phase of the scattered wave is expressed as

$$\delta\varphi = (\vec{k}_i - \vec{k}_s) \cdot \vec{d}$$

when the surface undergoes a translation of a quantity \vec{d} . The sensitivity of the phase to the surface translation is thus directly linked to the direction of displacement vector \vec{d} . When the incident and scattered wave vectors are almost normal to the surface of the object, only deformations perpendicular to the surface will be observed.

Q19- Are you in this configuration in your set-up?

2. Measurement of the deformation by a subtractive method

The treatment consists simply to subtract the image of the reference interferogram pattern before deformation, to the interferogram pattern obtained after deformation, to calculate

$$\Delta I_{CCD}(x, y) = I_{CCD}(x, y, \Delta t) - I_{CCD}(x, y, 0).$$

Q20- Give the expression of $\Delta I_{CCD}(x, y)$ showing that it is proportional to $\sin(\Delta\varphi(x, y, t)/2)$ and explain the principle of the subtractive method. What the obtained fringes do represent? What is the deformation of the piezoceramics? How can we deduce from the measurement the maximum amplitude of the ceramic deformation?

→ Acquire an image of the ceramics for a zero applied voltage, then the image for an applied voltage of some Volts.

→ Realize and display the absolute value of the difference between the two images.

<code>franges= abs(double(img_def)-double(img_ref)) ;</code>	<i>Realize the difference between two images</i>
<code>imagesc(franges) ;</code>	<i>Display the image in full scale and orthonormal mark</i>

→ Realize the measurement of the maximal amplitude of the ceramic deformation as a function of the applied voltage. You can use the following function, that realizes the image difference in real time.

<code>module_soustraction_reference(vid,N_itér);</code>	<i>Realize the difference of the current image with a stored reference image in real-time (N-itér. times)</i>
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Q21- What is the smallest deformation fringe spacing that we can distinguish? The ability to distinguish close fringes, means the ability to measure large deformations, deduce from this a constraint on the maximal size of the speckle grains.

3. Speckle filtering

It is possible to filter the speckle in the final deformation fringe pattern using a convolution filtering (for example using a convolution by a square matrix). Matlab has simple to use functions such as:

<code>img_filtree= filter2(fspecial('average', N),img);</code>	<i>Realize the convolution of the image with a square matrix of dimension N x N filled with 1</i>
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Be careful, the speckle being in the case of that image a multiplicative noise, that filtering will be efficient only after having made a nonlinear operation such as the absolute value, like in the "franges" function seen previously.

→ The script that performs this filtering (N has to be large compared to the average size of the speckle but small compared to the spacing of the deformation fringes) is giving below.

<code>filtrage_soustraction_reference(vid,N, N_itér);</code>	<i>Perform the difference of the current image with the reference image in real-time (N-itér times) and filters the speckle with a kernel of size N</i>
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V. Speckle interferometry – Study of the vibration modes of the object

For the vibration measurement, the object is sinusoidally excited at a frequency ν_s what gives us $\Delta\varphi(x, y, t) = \Delta\varphi_{Mode \nu_s}(x, y) \sin(2\pi\nu_s t)$. The amplitude $\Delta\varphi_{Mode \nu_s}(x, y)$ varies with the frequency of the vibration and reveals the spatial structure of the vibrations modes with its vibration nodes ($\Delta\varphi_{Mode \nu_s}(x_N, y_N) = 0$) and antinodes ($\Delta\varphi_{Mode \nu_s}(x_V, y_V)$ maximal). The period of the excitation being in general much more rapid than the response time of the CCD camera, the camera doesn't resolve temporally that excitation and temporally integrate the signal to finally measure the average value of the collected intensity $\langle I_{CCD}(x, y, t) \rangle_{\tau_{CCD}}$ on the integration time τ_{CCD} of the CCD camera. We can write that intensity as:

$$\langle I_{CCD}(x, y, t) \rangle_{\tau_{CCD}} = I_0(x, y) \left[1 + m(x, y) \cos(\varphi_0(x, y)) \langle \cos(\Delta\varphi_{Mode \nu_s}(x, y) \sin(2\pi\nu_s t)) \rangle_{\tau_{CCD}} - m(x, y) \sin(\varphi_0(x, y)) \langle \sin(\Delta\varphi_{Mode \nu_s}(x, y) \sin(2\pi\nu_s t)) \rangle_{\tau_{CCD}} \right]$$

or:

$$\langle I_{CCD}(x, y, t) \rangle_{\tau_{CCD}} = I_0(x, y) \left[1 + m(x, y) J_0 \left(\Delta\varphi_{Mode\ v_S}(x, y) \right) \cos(\varphi_0(x, y)) \right]$$

with $J_0(\varphi)$ the zeroth order Bessel function. On the vibration nodes $J_0(\varphi)$ is maximal with value 1, what means that the speckle structure keeps a high contrast, whereas for the vibration antinodes $J_0(\varphi)$ goes to 0 what leads to a scrambling of the interference pattern and a decrease of the contrast of the speckle pattern.

➔ You can obtain beautiful vibration modes structures for the piezoceramic by connecting it to the Low Frequency Generator and using the following command:

<code>filtrage_soustraction_vibration(vid,N,N_itér);</code>	<i>Perform the difference between two successive images in real-time (N-itér times) and filters the speckle with a kernel of size N</i>
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The previous measurements were essentially qualitative. They do not allow a direct measurement of the phase variation induced by the deformation or the vibration of the object. To extract $\Delta\varphi(x, y, t)$, we must couple the DSPI system with measurement techniques of the "phase shift" type (similar to the "phase shift" technique used in ZYGO interferometers). The previous command performs such a measurement (with two phase levels). It performs the difference between two successive images, and simultaneously applies an additional π phase between the two beams for each image acquisition (using the piezomirror placed in the object path).

Q22- Show that such a simple treatment allows to eliminate the average intensity term $I_0(x, y)$ and doubles the useful signal $J_0 \left(\Delta\varphi_{Mode\ v_S}(x, y) \right)$.

Homodyne Sensor & Heterodyne sensor

Answering the questions didn't replace a redaction work for the compte-rendu. These questions are essentially there to guide you for the labwork and for that redaction.

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The study of sonic and ultrasonic waves constitutes a particularly significant field of metrology and non-destructive testing. All objects submitted to a mechanical excitation, either internal as in an engine, or external as in the case of an ultrasonic excitation with a piezoelectric transducer, vibrate with a spectrum characterizing the object. An object's flaw defect will modify this spectrum. For example, a cracked dish plate emits a different noise when it is knocked. A sound study using a microphone can give a significant information on the physical integrity of an object (abnormal noise in an engine, or distorted note of a musical instrument). However, to be more precise this general study of the sound emission of an object (due to the vibration of the entire object) will have to be completed most of the time by a more particular vibration study of an amount number of points on the surface of the object in order to identify the special defective place. Supposing that the object vibratory behavior is not disturbed by the vibration sensor, that the number of studied points is limited and especially that these points of measurement are accessible, this study can be carried out using vibration sensors (like accelerometers) set on the object surface.

For all cases where the use of mechanical vibration sensors is impossible, optical methods offer interesting alternatives: they allow to carry out remote and fast measurement of the object vibrations. The principle of laser vibrometry is to send a laser beam on the object point in order to measure the amplitude vibration $\delta(t)$. The reflected beam is phase modulated. The amplitude of phase modulation is proportional to the vibration amplitude $\Delta\varphi(t) = 4\pi \delta(t)/\lambda$, where λ is the used wavelength. The problem is now to transform this unworkable phase modulation into a modulation of intensity, which can be measured by a photodiode. This transformation is carried out thanks to an interferometer in which the phase-modulated beam interferes with a reference beam.

The aim of this Practical Labwork is to study two interferometric devices of vibration measurement. The reference optical wave has the same frequency as the signal wave in the homodyne sensor. They have different frequencies in the heterodyne sensor (it is in fact a fixed difference frequency between these two waves). We will measure very low amplitude (typical order is the nanometer or tenth of nanometer) and high frequency (several hundred kHz even a few MHz) vibrations. So, the studied assemblies are particularly adapted to the study of ultrasonic waves.

HOMODYNE SENSOR

The homodyne sensor is a Michelson interferometer with mirrors assembled on piezoelectric transducers (Fig.1). M1 mirror piezoelectric transducer is supplied with DC voltage what allows to finely tune the path difference between the two beams. M2 piezo-mirror, that simulates the target to be measured, is supplied with a sinusoidal voltage. The source is a red laser diode emitting 3mW at 670nm. A polarizing beam splitter (PBS) Cube transmits the horizontal polarization components of the beam coming from the laser diode (i.e. in the plane of the figure) and reflects the vertical component.

The junction plane of the laser diode, that also defines the polarisation direction is oriented at 45° in order to equalize the intensity send in the two arms of the interferometer. On each arm the polarization is 90° rotated thanks to a double pass in a quarter wave plate. Thus, the beam initially transmitted by the PBS Cube is now reflected, whereas the one initially reflected is

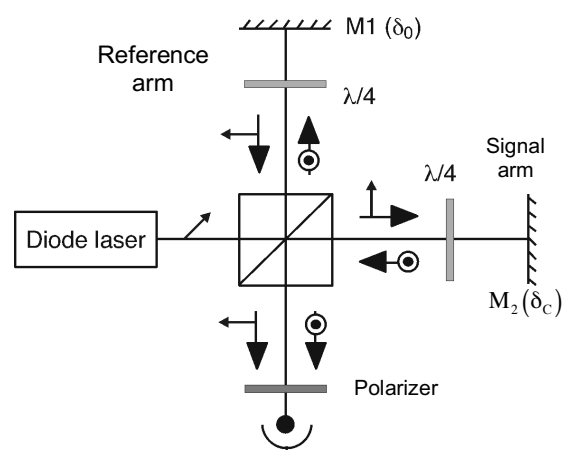


Figure 1 : Scheme of the experimental set-up for the homodyne detection sensor

now transmitted. A polarizer then allows the two orthogonally polarized waves to interfere on the photodiode.

I. Preparation of the Practical Labwork

The homodyne sensor corresponds on the optical point of view to a two-wave interference set-up between a signal beam of amplitude E_S (of frequency f_S) and intensity I_S and a reference beam of amplitude E_R (and of identical frequency $f_R = f_S$) and intensity I_R .

One notes $\Delta\varphi$ the phase shift between the two signal and reference waves:

$$\Delta\varphi(t) = \frac{4\pi}{\lambda} (\delta_0 + \delta_C \sin(2\pi \nu_C t))$$

where δ_0 is the displacement of mirror M1 (that allows to adjust the interference state at the detector level) where as **δ_C is the amplitude of the vibration of the target, i.e. of the mirror M2 modulated at frequency ν_C , that we want to measure.**

Q1- Supposing a perfect and perfectly aligned set-up, establish the theoretical expression of intensity I_{rec} received by the detector, as a function of the intensities I_S and I_R of the signal and reference beams and of the phase shift $\Delta\varphi$ (we will suppose for the calculations that we have two perfectly superposed plane waves).

Q2- We suppose that the displacement of the mirror that we want to measure is of very small amplitude ($\delta_C \ll \lambda$). Develop the received intensity expression to see directly terms proportional to δ_C and δ_C^2 . What is the frequency of the detected signal corresponding to these different terms?

1. Choice of operating point

The detector received intensity has several terms corresponding to the signal that we want to measure, or that will be useful to optimize and characterize the set-up.

Q3- Show that we can use the modulation $m = (I_{Max} - I_{Min}) / (I_{Max} + I_{Min})$ of the interference fringes and the mean intensity $I_0 = I_R + I_S = (I_{Max} + I_{Min}) / 2$ to write the intensity received by the detector as $I_{rec} = I_0 (1 + m \cos(\Delta\varphi(t)))$. For that, we have defined I_{Max} and I_{Min} the intensities of the bright and dark fringes (measured, by varying δ_0 , without applied signal, i.e. with $\delta_C = 0$). How do you choose I_R and I_S to maximize m .

Q4- What are the characteristics of the detected signal for $\delta_0 = 0$, $\lambda/8$ and $\lambda/4$. Deduce from it the value of δ_0 that optimizes the signal component detected at frequency ν_C ? This optimum operating regime consists in setting the reference and signal "in quadrature", explain the sense of this term. Write then the expression of the amplitude of the detected signal at frequency ν_C , relating it to the fringe modulation m and to the mean intensity I_0 .

Q5- Using a curve plotting I_{rec} as a function of δ_0 , make a scheme explaining the response of the system to a small amplitude vibration.

Q6- What happens when the amplitude δ_C of the vibration increases and that the condition $\delta_C \ll \lambda$ is not fulfilled anymore?

2. Conditions for a good operation of the set-up

The previous expressions are written in the case where the two-wave interference set-up is perfectly aligned. That means that the set-up is aligned until uniform tint, what imposes an alignment constraint on the orientation of mirror M1 compared to M2.

Q7- What do we have to do to align the set-up to uniform tint? Imagine an alignment process allowing to reach that uniform tint.

- Q8-** When the two mirrors make an angle α between them, we observe fringes in the plane of the detector. Write the expression of the fringe spacing as a function of α .
- Q9-** What should be the minimum grating spacing of the equal thickness fringes in order to have uniform tint for a detector of diameter d ? Deduce from it the maximum angle between the two mirrors for a detector of diameter $d=1\text{mm}$.
- Q10-** Do you see other important points for a good operation of that optical system.

II. Experimental part

→ Align and correctly adjust the optical set-up elements (mirrors, quarter wave plates, polarizer, photodiode) in order to observe fringes with a good contrast. Then obtain uniform tint. There is no necessity to apply a voltage on the M2 piezo mirror.

1. Calibration of the system

One of the advantages of optical vibration measurement systems is the possibility of knowing absolutely the amplitude of the measured vibration, using the length reference that is λ . **The calibration therefore consists in relating the amplitude of the measured signal ($V_C \sin(2\pi \nu_C t)$) to the amplitude of displacement ($\delta_C \sin(2\pi \nu_C t)$) which created this signal.** The calibration factor, given by the ratio ($K_C = \delta_C/V_C$), is generally expressed in (nm/mV).

Q11- The calibration factor depends only on the characteristics of the interference fringe system. From the preceding theoretical formulas give the expression of the detected signal, V_C at the excitation frequency, ν_C as a function of δ_C (as a function of δ_0 , m and I_0). What is it worth for phase shift δ_0 optimal?

Q12- Measure the fringes modulation m and the mean intensity I_0 .

Q13- Deduce from them the homodyne sensor calibration factor K_C in nm/mV.

2. Measurement of a vibration amplitude

We apply to the piezo-mirror M2 a signal of frequency around $\nu_C=120\text{kHz}$, and amplitude 1V (i.e. 2V peak to peak). The detector has a detection bandwidth of 7MHz. It is connected to high pass filter with cut-off frequency of 7kHz. That filter has two outputs: the first one is the filtered output (AC connector) and the second one is an unfiltered output (AC+DC connector), allowing to measure the value of the DC part of the signal simultaneously to the filtered signal.

Q14- Explain the interest in having those two outputs.

Q15- Measure the amplitude δ_C of the displacement of the mirror M2, using the previously determined calibration factor K_C .

3. Minimum detectable displacement

The ultimate quality of vibrations measurement devices is given by their ability to measure very small displacements. That ability is measured by the sensitivity S , defined as the minimum displacement giving a signal to noise ratio of 1 (i.e. 0 dB), for a detection bandwidth B of the detection device normalized to 1Hz:

$$S = \frac{\delta_{Cmin}}{\sqrt{B}}$$

That sensitivity is in general given in $\text{nm}/\sqrt{\text{Hz}}$. It allows an easy comparison between various systems generally operating with different measurement bandwidth.

Q16- Determine the smallest value of δ_C (minimum detectable amplitude δ_{Cmin}) that can be measured experimentally (and without signal averaging) at the excitation frequency of 120kHz. Compare the values obtained on the filtered output (AC) and on the unfiltered output (AC+DC).

Q17- Deduce from it the sensitivity of your set-up, taking into account its detection bandwidth. Why do we normalize to the squared root of that bandwidth?

When the measurement limiting noise is **the photon noise**, one can show that the theoretical sensitivity of the homodyne sensor for an optical power P (in Watt) of a beam of wavelength λ (corresponding to photons of energy $h\nu$) on a detector of quantum efficiency $\eta = 90\%$ is given by:

$$S_{BPh} = \frac{\lambda}{4\pi} \sqrt{\frac{h\nu}{\eta P}}$$

Q18- Calculate the theoretical sensitivity of your set-up and compare it to your measurement. What is then your conclusion on the origin of the noise limiting your measurement?

Q19- With the available means of the Labwork, what kind of solution could you propose to increase the experimental sensitivity, i.e. decrease S , by a factor of 10 (quantifying it)?

Subsidiary Question: The homodyne detection principle is used in gravitational wave detection systems (in a largely improved and optimized version). In these devices, we don't measure the vibration of the mirror but the strain $h(t)$ of the interferometer arm under the influence of the gravitational wave arrival, this strain inducing a change $\Delta L(t) = h(t) L$, in the arm length. Supposing that S_{BPh} gives the ultimate sensitivity of the interferometer, calculate that sensitivity for a LIGO/VIRGO type interferometer, using a wavelength of $1.06\mu\text{m}$, with a laser power of 10W and a quantum efficiency of 90%. What interferometer arms lengths L do we have to use to detect the arrival of a gravitational wave inducing a strain $h(t) = 10^{-23}(1/\sqrt{\text{Hz}})$. How do we proceed to reach this length in practice?

HETERODYNE SENSOR

The main limitation of the homodyne detection is its extreme sensitivity to low frequency noise, that usually add to the signal we want to measure. The operation of the homodyne detection usually requires a servocontrol that stabilizes actively the set-up to the optimum operating point. That servocontrol complicates the set-up, what is the reason why a derived set-up is generally preferred to the homodyne detection, **the heterodyne detection**, in which the two interfering beams are at different frequencies with a fixed frequency difference.

I. Analysis of the system

The two beams interfering in the interferometer, comes from the same He-Ne laser ($\lambda=633\text{nm}$) and have a frequency shift between them of $f_B \approx 70\text{MHz}$, introduced by an acousto-optic cell.

Q1- Show that this frequency shift induces a temporally linear phase variation, what allows to replace, in the expression previously established for the homodyne detection, the phase difference, δ_0 , between the two beams by $\lambda f_B t / 2$.

If the two beams interfere with a uniform tint, the measured photo-current will be sinusoidally modulated around the frequency f_B (the DC component of the signal being cut-off by the detection electronics it is not considered here)

$$i_{HF}(t) = k \cos\left(2\pi f_B t + \frac{4\pi}{\lambda} \delta_C \sin(2\pi \nu_C t)\right)$$

We can develop the signal as a Fourier Serie:

$$i_{HF}(t) = k \text{Re} \left[e^{2i\pi f_B t} \left(J_0\left(\frac{4\pi\delta_C}{\lambda}\right) + 2i J_1\left(\frac{4\pi\delta_C}{\lambda}\right) \sin(2\pi\nu_C t) + 2 J_2\left(\frac{4\pi\delta_C}{\lambda}\right) \sin(4\pi\nu_C t) \right) \right]$$

What will give finally, restricting the development to first orders:

$$i_{HF}(t) = k \left[J_0 \left(\frac{4\pi\delta_C}{\lambda} \right) \cos(2\pi f_B t) + J_1 \left(\frac{4\pi\delta_C}{\lambda} \right) (\cos(2\pi(f_B - \nu_C)t) - \cos(2\pi(f_B + \nu_C)t)) \right]$$

The displacement signal carried by the signal beam will be seen as lateral bands on both side of the beating frequency (or heterodyning frequency) in the photocurrent spectrum. The height of the different peaks is given by the Bessel function of order n with the argument $4\pi\delta_C/\lambda$ (Fig. 2). An electronic card makes a treatment of the photocurrent to extract the displacement signal and make it usable.

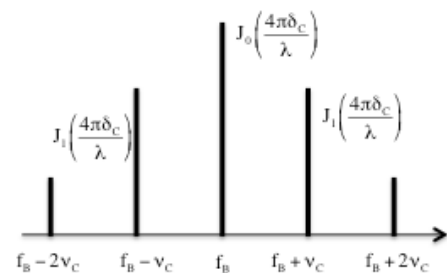


Figure 2 : Photocurrent spectra

When the displacement δ_C is very small compared to the wavelength, we have:

$$\frac{J_1(4\pi\delta_C/\lambda)}{J_0(4\pi\delta_C/\lambda)} \cong \frac{2\pi\delta_C}{\lambda}$$

Q2- What is then the attenuation in dB between the beating signal power (central peak) and the signal power (lateral peak) for a displacement amplitude of 1nm?

A detailed analysis of the working principle of the heterodyne detection is given in the **documentation BMI** that can be find in the Labwork room or on the Libre-savoirs site.

Remark: the optical axis of the lens focusing light on the vibrating target is parallel but not superposed system optical axis. The interest of such a misalignment is that it prevents back reflection to the laser that would disturb its operation.

II. Alignment of the vibrating target

The target is not the M2 piezo-mirror used in the first part (its characteristics, particularly its resonance frequencies will then be different). For the alignment, it is not necessary to apply a voltage to the piezo-mirror.

→ Put the command switch on "Manuel" and put the preamplifier button on "30", and turn the potentiometer on the position "Zero"

→ Align mirror M2 in order to superimpose the signal beam with the reference beam and taking care to correctly send those two beams on the detector, optimize the beating signal due to interference of two beams on the detector (use the HF output on the heterodyne sensor box, connected to the oscilloscope in order to measure the beating oscillation at 70MHz), progressively decreasing the preamplifier gain down to "0".

→ If the target is correctly aligned, the Automatic Gain Control must operate when switching the switch from "Manuel" to "Automatique" (red LED on).

III. Study of the raw signal on the spectrum analyser

→ Apply on mirror M2 a voltage of frequency $\nu_C=200\text{kHz}$, and amplitude 1V (i.e. 2V peak to peak), and the HF output is connected to the electrical spectrum analyser.

Q3- Given the characteristics of the expected spectrum for the electrical signal delivered by the HF channel (Fig. 2), what setting should you take to:

- the center frequency of the display "Freq"?
- the spectral range of measurement "Span"?

For the "BW" bandwidth, you will take a value of 1kHz (or 300Hz) for the "RBW" resolution bandwidth with the video bandwidth "VBW" in automatic mode "VBW auto", and for the amplitude "Ampt" the automatic mode "Auto Range".

Q4- Measure δ_C on the electrical spectrum analyzer from the raw signal given by the photodiode using the H.F. output of the probe. For this, you can use the markers with the functions "Mkr" to display them and " Mkr ▶ " to position them on the peaks. The

indications of measurements of the lateral peaks are either in absolute (symbol " m_i ") or in relative (symbol " d_i ") with respect to the central peak (peak 1), mode selectable with the function "Marker type".

Q5- Give the experimental value of the minimum detectable vibration amplitude.

Q6- What is the detection bandwidth of that new set-up? Deduce from these measurements the sensitivity S_{AS} of the heterodyne detection for the raw signal obtained directly at the detector output and analysed with the spectrum analyser.

IV. Study after demodulation electronic processing

The spectrum analyser, thanks to its measurement principle, allow to measure the amplitude of the displacement, but that measurement is limited to periodic signals that are stable on the relatively long measurement duration. Unfortunately, most of the time practical ultrasonic signals, that are to be measured with heterodyne sensors, consists in series of non periodic echoes or in unique non repetitive signals, which means that the spectrum analyser can not be used for the measurement. The raw signal has to be electronically processed in order to be exploitable. That electronic signal processing system will transform the beating signal with its lateral band to an electrical signal directly proportional to the object displacement (in the case $\delta_c(t) \ll \lambda$). That signal after electronic processing is available on the BF output of the control box of the heterodyne sensor. The operation principle of the demodulation is available in the "**documentation BMI**" of the heterodyne sensor.

→ Apply to the M2 piezo-mirror a sinusoidal signal of frequency greater than 200kHz.

→ Use simultaneously the BF output on the oscilloscope and the HF output on the spectrum analyser to give a correspondence between the voltage amplitude detected on the oscilloscope and the amplitude of the vibration $\delta_c(t)$ of the mirror measured according to the technique of paragraph III.

Q7- Give the results of that calibration in mV per nanometres.

Q8- Verify the calibration for some frequencies between 200kHz and 4MHz.

Q9- Measure the minimum measurable displacement and the sensitivity, taking into account that the bandwidth of the processing system goes from 200kHz to 4MHz.

Q10- Compare the sensitivity obtained using the different methods (homodyne detection, heterodyne detection used with spectrum analyser, heterodyne detection used after demodulation electronic processing).

V. Measure with a scattering target

Delicately put a scotch tape on the piezo-mirror to simulate a scattering target.

Q11- Make a measurement with this scattering target. Comment the obtained results.

D 3

Diffractive Optical Elements (*DOEs*)

Version: June 30, 2021

Questions P1 to P7 have to be prepared before the labwork.

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Introduction

The rapid development of photolithography techniques today allows the fabrication of phase masks that are more and more precise and complex. These phase masks (*DOE : Diffractive Optical Elements*) allow, by a very precise modification of the optical wavefront, to obtain the expected intensity pattern after propagation of laser beams.

Identically the development of spatial light modulators (*SLM : Spatial Light Modulator*) and of electromechanical micro-systems (*MEMS : Microelectromechanical systems*, such a micro-mirrors) allow to control locally the phase of the wavefront and to act on the light beams in real time.

All these components (*DOE*, *SLM*, *MEMS*, deformable mirrors) are used in an increasing number of cutting-edge applications. Example of applications of DOE or of SLMS :

- laser beam forming ,
- microscopy, biology, biophotonics (optical tweezers, molecule manipulation),
- atom optics (cold atoms manipulation),
- instrumental optics (aberration correction, adaptive optics, displays, structured illumination, calculated diffusors, microlenses arrays...).

In the labwork room, you will find an example of *DOE* manufactured by the company *Holotetrix*. Illuminated by a HeNe laser, you will be able to see the quality of the obtained structured illumination patterns and the quasi-disparition of the 0 diffraction order.

Here are some website of some manufacturers of these various devices (actives ou passives):

sales.hamamatsu.com/assets/pdf/parts_X/x10468_series_kacc1172e06.pdf

www.holoeye.com

www.itrc.narl.org.tw/index-e.php

During this labwork, we will realize some example of Diffractive Optical Elements. The used support will be a simple slide printed in Black and White by a laser printer. Precisely, we will realize *DOE* printed in Black and White, which Optical Fourier Transform will allow to obtain the expected intensity pattern in the Fourier plane.

We will see several methods to calculate these *DOE* all based on calculation of discrete Fourier Transform (FFT).

1 Preparation to do before the Labwork

The principle of the set-up used during this labwork is given by the figure [3.1](#) below :

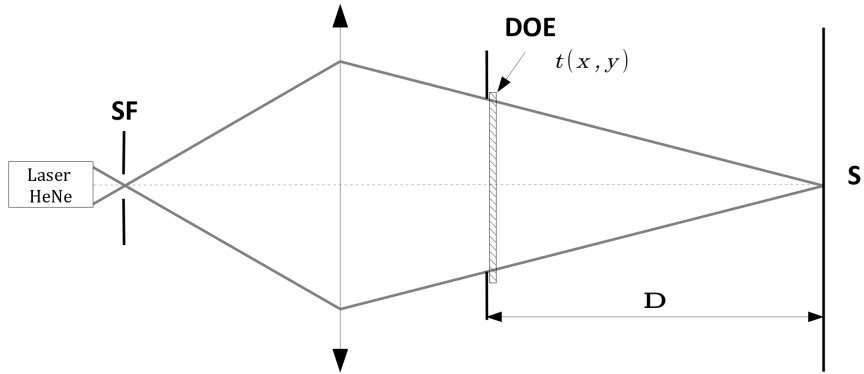


Figure 3.1: Optical set-up, SF: Spatial Filtering, DOE: Diffractive pupil, S: Screen (or CCD camera)

This is the set-up seen during first year for the Diffraction at infinity Labwork. The HeNe laser is equipped with a filtering system (microscope objective $\times 20$ and a $20\ \mu\text{m}$ hole) allowing to uniformly illuminate the optical system (here a simple achromat doublet) that makes the conjugation of the source hole with the image plane. The Pupil-Image Plane distance is $D \approx 3\ \text{m}$ (limited by the physical dimension of the labwork room).

In the plane of the pupil, we will position a diffractive element of known complex transmission $t(x, y)$.

P1 How do we calculate the intensity pattern in the image plane ?

P2 What is the intensity pattern and the image spot size if $t(x, y) = 1$ on the whole pupil (and 0 outside the pupil) ?

P3 What is the intensity pattern if the diffractive element is a simple amplitude sinusoidal grating of period T_R :

$$t(x, y) \propto \left(1 + \sin \left(\frac{2\pi x}{T_R} \right) \right)$$

What is the relative amplitude of orders $+1$ and -1 compared to order 0 ?

P4 Same question if the diffractive element is a Foucault resolution target with the same period. Reminder : a Foucault resolution target is a binary grating :

$$t(x, y) = 0 \text{ for } x \text{ comprise between } 0 \text{ and } \frac{T_R}{2}$$

$$t(x, y) = 1 \text{ for } x \text{ between } \frac{T_R}{2} \text{ and } T_R$$

P5 Why is it not possible to obtain an order 0 with a zero amplitude if $t(x, y)$ is a real transmission ?

P6 Give an example of a periodical diffractive element allowing to obtain an order 0 with a zero amplitude.

We will, in the second part of the labwork, calculate and realize a *DOE* that will allow by an optical Fourier transform to obtain your expected image (your "œuvre d'art").

P7 This expected image is a binary image (Black and White) that you will have to prepare with any software (for example `Paint`). Choose :

- Image attribute : 128 by 128 pixels,
- Color : Black and White.

Do a (free !!!) drawing, preferentially simple and more importantly **Black on a White Background**. It is also advisable to frame the drawing in black, to precisely delimit the object and not to draw too fine lines. Save this file in a BMP format on the USB you will brought with you for the labwork.

2 Correction of an aberration by a diffractive optical element

Optical system aberration correction by *DOE* is a very active domain (for example : the correction of chromatic aberrations in the infrared domain). You will study here a *DOE* example that will allow to correct a geometrical aberration.

2.1 Principle of the correction of an aberrant wavefront

A geometrical aberration results of a phase defect of the real wavefront compared to the spherical wavefront that would be given by a perfect optical system. Let's note $\Phi_{\text{aberration}}(x, y)$ this aberrant phase.

To correct this defect, it is enough in principle to have an optical system able to compensate exactly this phase defect with the opposite sign $-\Phi_{\text{aberration}}(x, y)$. We can use for example a deformable mirror, a liquid crystal matrix or a photorefractive crystal (index variation) and realize thus an active (static correction) or adaptive (dynamic correction) optics. All these devices are usually expensive and it is impossible to phase modulate the wavefront with the material used for the labwork. We will overcome this difficulty by using a carrier

with a sinusoidal transmission that will be phase modulated. We can thus realize a very simple experiment with a computer generated *DOE* printed with gray levels on a laser printer slide.

The phase that we want to correct $-\Phi_{\text{aberration}}(x, y)$ is coded thanks to a variable spacing sinusoidal amplitude grating :

- The sinusoidal grating playing the role of the carrier has a transmission that is :

$$t(x, y) \propto (1 + \sin(2\pi f_R x))$$

where f_R is the spatial frequency of the grating in mm^{-1} ,

- The aberrant phase, $\Phi_{\text{aberration}}(x, y)$, to correct is coded as a modulation of the phase of this carrier. We then obtain a distorted grating, which transmission factor is :

$$t(x, y) \propto (1 + \sin(2\pi f_R x - \Phi_{\text{aberration}}(x, y)))$$

2.2 Implementation

↪ In order to realize the simulation and the printing of the diffractive pattern, use in `Matlab` the script `>>diabholo`. The aberrant phase is calculated with the Zernicke polynomes (see for example the expressions in the ZYGO documentation). It is possible to choose the grating spacing and the number of points used for the calculation, and then to choose an aberration among the 3^{rd} order aberrations or corresponding to a Zernicke polynome of higher order.

The program shows successively :

- the perfect sinusoidal grating ,
- its 2D FFT,
- the modulated grating,
- its 2D FFT,

Q1 Interpret the gratings and their 2D FFT obtained for various aberrations and various value of the grating spacing. For high value of the aberration explains the aliasing phenomenon observed on the 2D FFT.

↪ Choose a coma or astigmatism aberration. Calculate the corresponding grating and print it on the slide.

↪ Do the same thing for a large defocus.

↪ Realize the optical Fourier transform set-up of the figure [3.1](#). Verify, and if necessary align the laser cleaning system. The Clairault has to be rigourously used on its axis and in its good orientation, realize the necessary alignment. Realize the precise focus on the scattering screen.

↪ Use in a first step the *DOE* corresponding to the defocus defect.

Q2 Give the expression of the intensity in the Fourier plane as a function of the aberrant phase and of the set-up parameters in the case of a defocus defect.

Q3 Observe and comment the figure obtained in the Fourier plane. Verify that we effectively obtain a defocus defect in the $+1$ and -1 diffraction orders. Evaluate the value of the defocus defect and compare it to the expected value considering the amplitude of the aberrant phase you choose.

↪ Then place the *DOE* corresponding to the aberrations you choose.

Q4 Observe and comment the pattern you obtain in the Fourier plane.

Q5 Explain how these holograms can be used to correct the corresponding geometrical aberrations.

Q6 Compare the figure obtained with the simulations. Explain the presence of diffraction orders other than 0 , $+1$ et -1 orders. Use the microscope to help you to respond to this question.

3 *DOE* obtained through a Lohmann coding

The objective of this part is to realize a *DOE* that will allow you to visualize in the Fourier plane your "œuvre d'art" that we will call $O_{\text{Art}}(x, y)$. In order to do this, the first step consists in calculating the Fourier transform of that drawing: $\widetilde{O_{\text{Art}}}(x', y') = \mathbf{TF}[O_{\text{Art}}(x, y)]$. We will then see that it will be necessary to add to your drawing a random phase before being able to exploit its Fourier transform. That Fourier transform being in general a fonction with complex values $\widetilde{O_{\text{Art}}} \in \mathbb{C}$, it is necessary to obtain the expected diffractive element to code both the amplitude and the phase of this function. You will see that the Lohman coding is a very clever way to realize it.

3.1 Calculation of the 2D Discrete Fourier Transform

↪ Launch a Matlab session, then the `menu_HSO_2008`. program.

↪ The `Charger un dessin` menu allow you to open your œuvre d'art and to display it in Black on a White background as well as in negative (white on a black background). This negative will correspond to the intensity pattern that will be expected in the image plane.

↪ Realize the numerical Fourier transform (FFT) of your œuvre d'art.

Q7 Does the obtained results correspond to your expectations ? Why is the spectrum symmetrical around the zero frequency ?

One can see that the intensity has a very bad repartition in the Fourier plane. It will thus be very difficult to code it correctly taking into account the low number of coding levels we have at our disposition.

3.2 Addition of a random phase on the object

The clever solution to get around that difficulty consist in adding a random phase on the object.

↪ Use for this the menu `Ajouter une phase aléatoire`. That function realize the following operation :

$$O_{\text{Art}}(x, y) \implies O_{\text{Art}}(x, y)e^{i\phi(x,y)}$$

where $\phi(x, y)$ is a uniform random variable varying between $-\pi$ and $+\pi$.

↪ Calculate again the FFT.

Q8 Why will the spectrum obtained when adding the random phase be easier to code ? Why will the object, after inverse FT, be correctly recovered despite this random phase ?

↪ Calculate the inverse Fourier transform to convince you that we really recover the unchanged object with this operation.

⟶ **In reality, the reconstruction of the object on which we add a random phase will lead to the presence of speckle on the reconstructed object.**

3.3 Principle of Lohmann coding

To restore your œuvre d'art from its 2D FFT, we need a way to code that FFT in modulus and in **phase**.

The very clever coding solution used here is due to Lohmann (*see the articles from Lohmann Applied Optics* 5, 967 (1966) and 6, 1739 (1967)) and is based on the use of the Huygens principle.

Your œuvre d'art is in fact a numerical image of 128×128 pixels :

$$O_{\text{Art}} [i, j] \quad i, j \in [1, \dots, 128]$$

1. One calculate the Discrete Fourier transform (FFT) of this image of 128×128 pixels :

$$O_{\text{Art}} [i, j] \xrightarrow{\text{FFT}} z_{n,m} = A_{n,m} e^{j\varphi_{n,m}} \quad n, m \in [1, \dots, 128]$$

2. One consider 128×128 elementary square cells with side dimension a . Each cell will code a complex number $z_{n,m}$ using the following method :

- (a) **Modulus coding.** Each cell contains a white rectangle which surface is proportional to the amplitude. The width of the rectangle is fixed (equal to $\frac{2a}{5}$), This is the height of the rectangle that is proportional to the amplitude.

$$h_{n,m} = a \cdot \frac{A_{n,m}}{A_{\text{max}}}$$

where A_{max} is the maximum amplitude in the 128×128 pixels.

- (b) **Phase coding.** The position of the white rectangle is horizontally shifted, compared to the cell center proportionally to the argument of the complex number.

$$c_{n,m} = \frac{1}{2} \cdot a \cdot \frac{\varphi_{n,m}}{\pi} \text{ avec } \varphi_{n,m} \in [-\pi, +\pi]$$

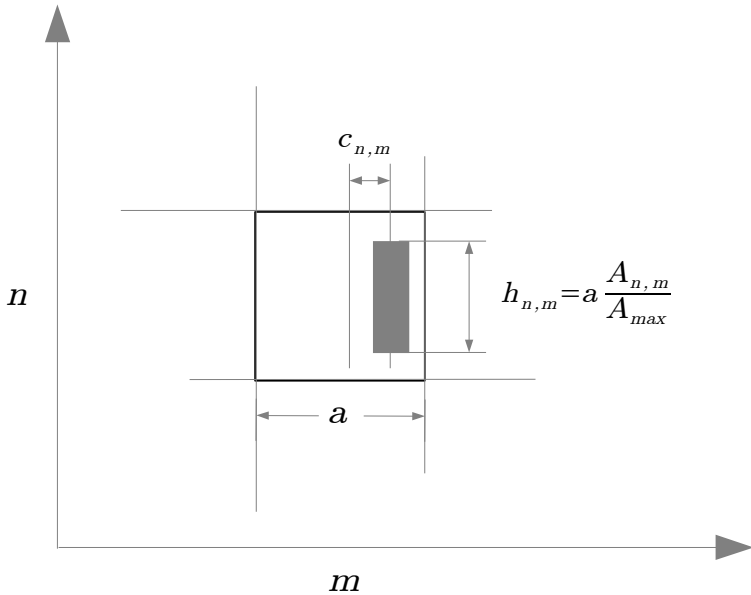


Figure 3.2: Principle of Lohmann coding

Q9 Explain simply the principle of the phase coding. What is the required position of the white rectangle in order to obtain $\varphi = 0, \pi, -\pi$. Which direction of the phase, of the recovered wavefront, will be correctly coded ?

Printing

↪ Print the Lohmann coding of the FFT in white on a Black Background, on a slide. Use the option `print` of the figure Matlab. The menu page setup allow to displace the hologram in the page and thus to economize the slides. Be sure that the printer uses its maximum resolution (1200 dpi).

Q10 Why, from a practical point of view, do we have to print a small size image with maximal dimension of the order of 2 to 3 centimeters ?

Q11 The printer resolution is 1200 dpi *Dots Per Inch* and an inch corresponds to about 2.54 cm. The printed figure has a side size of 2.71 cm. How much points do we have to code each cell of the hologram. Is this resolution sufficient ?

↪ Use the menu `Reconstruction Hologramme` to have an idea of the results of the Fourier transform that will finally be obtained. The amplitude in the Fourier plane is obtained by a 2D FFT on 1024 par 1024 pixels (*zero padding*). The intensity is on a logarithmic scale with a -40dB threshold.

3.4 Optical recovering of the image

One then need to realize an optical Fourier transform to recover the object.

↪ Observe the image (l'œuvre d'art?) recovered in the Fourier plane on the scattering screen.

Q12 Comment on the observed phenomenon. Are they compliant to your predictions? Why do you obtain several replica of the object ?

Q13 Why is the intensity of the recovered object not uniform ? Why is the intensity more important in the center ?

4 Coding of the real part of the object Fourier transform

Another clever solution to obtain the *DOE* consists in coding only the real part of the Fourier transform of the expected object.

↪ Use for this operation the menu

`Codage de la partie réelle de la FFT`. The coding is a binary coding that you can print on the slide as previously.

↪ Use the menu `Reconstruction Hologramme` in order to have an idea of the results of the optical Fourier transform that will finally be obtained.

Q14 Why do we obtain in that case the object and a symmetric replica of it ? What simple tricks do we used in order to prevent the object and it symmetric replica to be superimposed ?

5 Analysis of a phase DOE

↪ On the second set-up, place the "Nemo" DOE and align the set-up in order to obtain a clear image on the screen.

Q15 Is the image symmetric ? periodic ? pixellised ? What can be deduce on the DOE ?

↪ Observe the *DOE* with the microscope. Put the DOE on the deck without fixing it with the clamp in order not to scratch it !

Q16 What are its main characteristics ? How much levels have been used to engrave it ?

D 4

Spatial Light Modulator

At the end of this labwork session you will be able to :

- use a SLM for imaging (liquid crystal screens)
- use a SLM as a spatial modulator of the phase of a laser beam.

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1 Introduction

↪ Place a polariser in front of the screens at your disposal in the room (the 2 screens of the computer, your smartphone,...) and turn it.

Q1 Is the light from a liquid crystal screen polarised ?

The polarisation is the characteristic of light which is modified by a liquid crystal. A spatial light modulator is constituted by a matrix of liquid crystal pixels. Each pixel is driven by a specific voltage and can therefore independently modify the polarisation of the light wave passing through it.

Depending on the setup in which the device is inserted, this polarisation control can be used for different applications.

During this session you will use a spatial light modulator (SLM) for two kind of applications: the first one is the "amplitude modulation" and is used for imaging, and the second one is the "phase modulation" for the realisation of diffractive masks (holograms, *DOE Diffractive optical elements*).

2 Illuminance modulation. Imaging.

2.1 Experimental setup

During this session we will use the SLM *Holoeye - LC2002*, and a part of its datasheet is given on figure 4.1. It is driven via a VGA signal by a computer interface developed under *Matlab*.


	Display	
	Type:	SONY LCX016AL-6
	Colours:	Grey-level image playback
	Active Area:	26.6 mm x 20.0 mm (1.3")
	Number of image pixels:	832 x 624
	Pixel pitch:	32 μm
	Image frame rate:	max. 60 Hz
Contrast ratio:	typically 200:1	

Figure 4.1: Datasheet part of the SLM *Holoeye - LC2002*

This interface allows to send an image on the SLM and to take back the observation CCD Camera signal. It allows to transform the image to project signal (coded as grey levels) in command voltage applied to the SLM, according to a programmable transfer function (in an equivalent way to a contrast or brightness adjustment on usual screens). In our case this transfer function had been optimized to realize in an optimal way the various experiments you will have to do during the labwork. The image grey level values (which relation to the actual applied voltage is thus masked by the program) will thus be used as a reference all along the labwork.

This SLM works in transmission, and is inserted in the optical bench between elements which allow polarization control :

- at the input, a polarizer followed by a half-wave plate,
- and an analyzer allowing the selection of a polarization direction at the output.

The bench is presented on the photo in figure 4.2. A first lens (L_1) allows beam collimation, and a second one (L_2) allows to image the SLM on the plane of the observation camera.

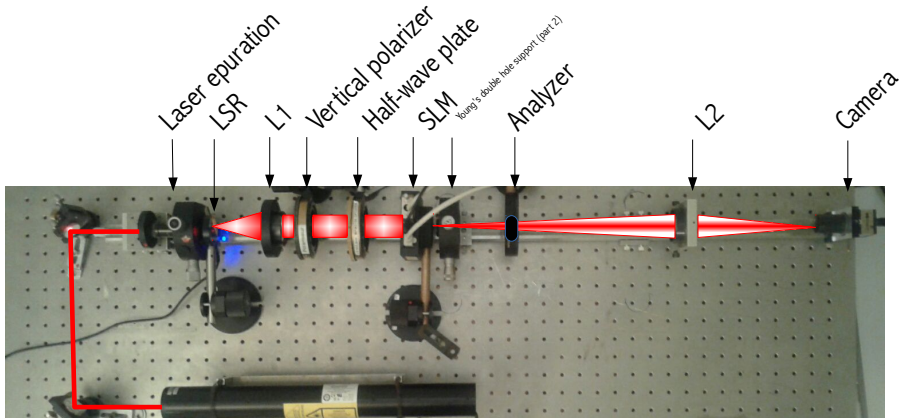


Figure 4.2: The different devices in the setup. Imaging configuration.

2.2 First observations

The figure 4.3 describes the configuration of the bench, used here to produce amplitude modulation.

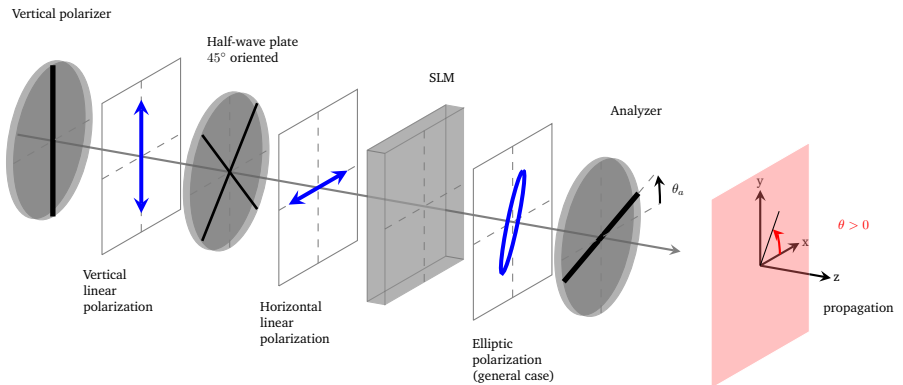


Figure 4.3: Imaging configuration. Orientations of the devices on the bench.

↪ Switch on the laser source and adjust the axis of the half-wave plate, placed after the vertical polarizer, in order to have a horizontal rectilinear polarisation at the input of the SLM (the polarizer and wave plate axes corresponds to the zero graduation).

↪ Place the analyser axis in the horizontal position ($\theta_a = 0$ sur la figure 4.3).

↪ Without running the interfacing software (or switching-off the SLM alimentation), visually evaluate the set-up transmission. The SLM pixels use the twisted nematic crystal technology (see the annex), deduce from your observation the "twist" angle of the nematic liquid crystal when the applied voltage is zero.

↪ Switch-on the SLM. Run the interfacing software for the control of the SLM and of the camera, and initialize the camera. For that purpose, run the software `matlab` and type `»SLM` in the command line, then select the menu `Initialisation Caméra`.

↪ With the help of the menu `Coupe Verticale`, apply a minimum grey level (NG=0) on one half of the surface of the SLM, and a maximal grey level (NG=255) on the other half.

↪ Using the menu `Paramètres Caméra`, adjust the camera exposure time to a value of about 0,2 ms.

↪ Adjust the position of the lens L_2 in order to image the SLM on the camera with an image magnification lower than 1.

Q2 Do the pixels driven with a zero grayscale level (NG=0, which correspond to the black pixels in the command image) appear in black on the camera? What can you then deduce on the applied voltage on the SLM for NG=0?

↪ Turn the analyser of 90° .

Q3 Explain the image modifications.

↪ Load a command image of your choice and check the result obtained on the camera as a function of the position of the polariser axis. For this purpose, use the menu `Charger une image`.

2.3 Contrast measurement

↪ Load again the two-part image with the menu

. Select the menu .

↪ Draw the curves of the average signal received by the camera as a function of the position of the polariser axis, for the command grayscale levels NG=0 and NG=255.

Q4 What is the position of the polariser axis which allows a maximal contrast ? And the position allowing the most luminous image for NG=0 ? For NG=255 ?

Q5 Explain the shape of both curves. Is the polarisation at the SLM output rectilinear ?

Q6 Give the expression of the polarisation ellipticity as a function of your measured curve parameters. Determine the orientation and ellipticity of the polarisation at the output of the SLM for a grey level command at NG=0 and at NG=255, from your experimental results. Compare your value to the value determined directly by the software.

2.4 Grayscale level linearity

↪ Place the analyser in the orientation allowing a maximal contrast, and draw the curve of the signal from the camera as a function of the grayscale level command applied to the pixel.

Q7 Is this curve linear ? What kind of curve would you get if the SLM only modifies the polarisation orientation as a function of the grayscale level command ?

This part illustrated the "amplitude modulation" operation of a SLM. The following part concerns the "phase modulation".

3 Phase modulation

3.1 Modifications of the optical bench

The figure describes the configuration of the bench, used here to produce phase modulation.

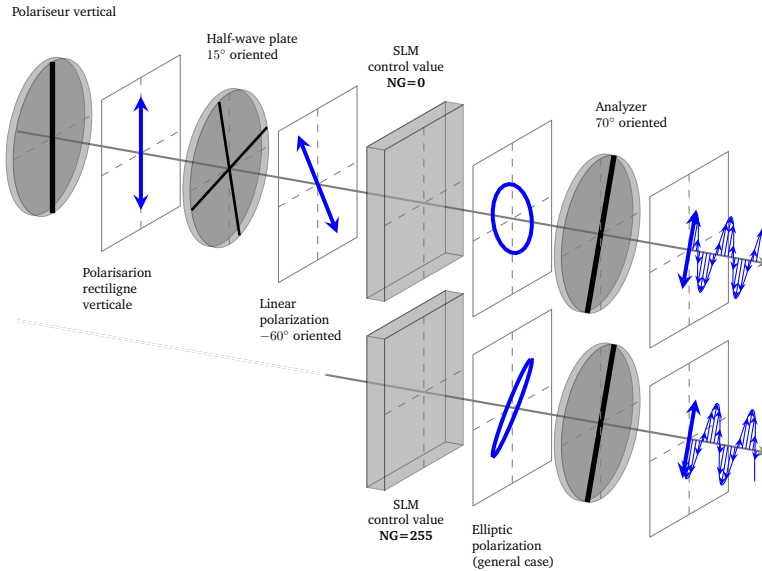


Figure 4.4: Phase modulation configuration. Orientations of the devices on the bench. Phase modulation principle.

Modification of the input polarisation and of the analyser axis orientation at the output

↪ Apply again zero grey level (NG=0) on the half of the SLM surface, and a maximal grey level (NG=255) on the other half, using the menu `Coupe Verticale`.

↪ Turn the half-wave plate in order to have a rectilinear polarisation input oriented along $\theta_p = -60^\circ$ (in other words, the axis of the half-wave plate should be oriented along an axis at $-30 + 45 = 15^\circ$). Place the neutral axis of the analyser along the position $\theta_a = 70^\circ$.

Q8 Why are the amplitudes so close in both zones in the image from the camera ?

The mystery device : LSR

↪ Remove the device "LSR" from the beam. Modify the exposure time of the camera in order to avoid image saturation.

Q9 This device is a "Laser Speckle Reducer". It is a scattering and vibrating plate. Explain the modifications observed on the image.

Diffraction

↪ Move the lens L_2 in order to place the camera in the Fourier plane of L_2 . And place a uniform command image. Set the exposure time of the camera to the minimum.

Q10 Why do several diffraction peaks appear in the horizontal and vertical directions on the image of the camera ?

↪ Measure the spacing between these peaks as a number of camera pixels.

The sensor of the camera is a matrix of 1280×1024 pixels, each pixel being a square of side $5.2 \mu\text{m}$ (that is to say a $6.6 \times 5.3 \text{mm}$ sensor). The lens L_2 is a Clairault of focal length $f' = 120 \text{mm}$.

Q11 Give the value of the spacing between the peaks in mm. Deduce an estimate of the SLM pixels size and compare it with the manufacturer's datasheets.

3.2 Phase shift calibration

Q12 What kind of setups allow the measurement of a light wave phase ?

In this part, we insert Young's double hole in the optical bench after the analyser, and we observe the fringes obtained in the Fourier plane.

↪ Place Young's double hole on the bench, after the analyser, and place the two-part command image with the menu Coupe Verticale.

↪ With the menu Paramètres Caméra, adjust the exposure time in order to avoid image saturation. Use in the same menu the tag `Size` in order to zoom in on the interference fringes (choose for instance a region of interest of 300×300 pixels starting from pixel $(450, 400)$).

↪ Adjust the position of the separation between both zones in such a way that it lies between the two holes. Check the good positioning of this separation by inverting the grayscale levels of both zones: the interference fringes must be shifted after this image inversion.

Q13 Explain why the fringes are shifted in such a way.

↪ Estimate the order of magnitude of this shift.

Q14 What can be deduced concerning the order of magnitude of the phase shift introduced by a pixel driven at $NG=0$ with respect to a pixel driven at $NG=255$?

↪ Observe the fringes profile using the menu `Profil de l'image` and modify if required the exposure time in order to avoid a saturation of the signal.

↪ Measure the phase shift introduced by the SLM as a function of the grayscale level command using the option `Calibration du déphasage`.

The option `Calibration du déphasage` determine the spatial shift of the interference pattern by fitting the fringes with a sine. It allows to determine both the amplitude of the fringes and the offset.

Q15 What is the value of the maximal phase shift obtained in this configuration ? Is it compatible with your first observations ? Is the phase shift linearly evolving with the command signal ?

Q16 Would the phase shift be the same for a different source, like for instance a laser at 532 nm ?

In this particular configuration, the SLM is therefore a tool for the phase modulation of a wavefront: the phase of each pixel is driven by a grayscale level command. It is this tool which is used in the following part.

3.3 Phase mask. Diffraction patterns

↪ Remove the Young's double hole from the setup and adjust again the exposure time in order to avoid image saturation.

↪ Apply a rectangular grating phase mask and vary both the grating pitch and the total phase shift. Replace the rectangular profile by a "sawtooth-like" one (en "dent de scie").

Q17 Analyze the different diffraction patterns obtained for these different gratings.

Q18 What is the relationship between de phase shift in the SLM plane $\Delta\phi_{\text{SLM}}(x, y)$ and the intensity $I(\sigma, \mu)$ in the Fourier plane ?

Realizing a phase synthetic hologram consists in finding a phase shift law $\phi_{\text{SLM}}(x, y)$ in order to obtain the desired intensity $I(\sigma, \mu)$ in the Fourier plane.

Q19 Why a simple inverse Fourier transform would not allow to calculate $\phi_{\text{SLM}}(x, y)$?

The following section presents a method for obtaining the desired mask: the Gerchberg-Saxton iterative algorithm.

3.4 Synthetic holograms.

Observations

↪ Draw a picture in white on a black background. Save it in monochrome Bitmap format (you can use the software `Microsoft/Paint` for instance).

↪ Calculate the corresponding phase mask using the option `Hologramme synthétique` and apply it to the SLM. About ten iterations of the algorithm are usually sufficient.

Q20 Analyse the diffraction pattern obtained in the Fourier plane. Is it close to what is expected ? What are the differences ?

Gerchberg-Saxton Algorithm

The calculation of the phase mask by the option `Hologramme synthétique` uses the Gerchberg-Saxton iterative algorithm. The principle of this method, published in 1972¹ is described on figure 4.5

Let us note $I_{\text{target}}(\sigma, \mu)$ the illuminance desired in the Fourier plane.

First step The calculation is started with an image characterized by a uniform amplitude and a random phase :

$$M_0(x, y) = e^{j\phi_0(x, y)}, \text{ where } \phi_0(x, y) \text{ is uniformly distributed between } 0 \text{ and } 2\pi.$$

¹R. W. Gerchberg and W. O. Saxton, "A practical algorithm for the determination of the phase from image and diffraction plane pictures", *Optik* 35, 237 (1972)

We then calculate the Fourier transform of this image, which is a complex quantity :

$$M_0(x, y) \xrightarrow{TF} |\widetilde{M}_0(\sigma, \mu)| \cdot e^{j\varphi_0(\sigma, \mu)}$$

From this quantity, we retain only the phase term and we associate to it the modulus required to construct the first numerical 'chimera' of the algorithm :

$$\widetilde{C}_1(\sigma, \mu) = \sqrt{I_{cible}(\sigma, \mu)} \cdot e^{j\varphi_0(\sigma, \mu)}$$

The inverse Fourier transform of this "chimera" gives another complex quantity whose phase will be a first approximation of the phase mask :

$$\widetilde{C}_1(\sigma, \mu) \xrightarrow{TF^{-1}} C_1(x, y) \cdot e^{j\phi_1(x, y)}$$

$\phi_1(x, y)$ is the phase mask calculated after this first step.

Recurrence This first phase mask is then used to construct a new image of constant amplitude :

$$M_1(x, y) = e^{j\phi_1(x, y)}$$

and a protocol identical to what was done for the initialization is applied :

Starting image		$M_1(x, y) = e^{j\phi_1(x, y)}$
TF calculation	→	$\varphi_1(\sigma, \mu)$
chimera construction		$\widetilde{C}_2(\sigma, \mu) = \sqrt{I_{cible}(\sigma, \mu)} \cdot e^{j\varphi_1(\sigma, \mu)}$
inverse TF calculation	→	phase mask $\phi_2(x, y)$

Iterations $\phi_2(x, y)$ is re-injected instead of $\phi_1(x, y)$ in the previous calculation to perform one more iteration, etc.

This recursive algorithm is described in figure [4.5](#)

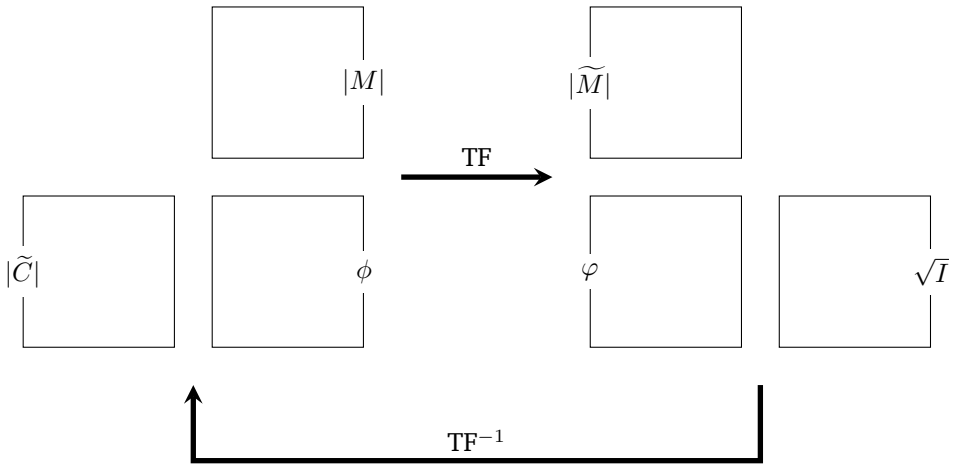


Figure 4.5: Principle of one iteration of the Gerchberg-Saxton algorithm. The phase mask ϕ is modified at each iteration.

Annex: Liquid Crystals Physics

Liquid crystals present a structure which associates both the properties of a crystal and those of a liquid. These properties were discovered at the end of the XIXth century and the first screens using liquid crystals pixels date from the 1970s.

Structure of a nematic pixel

In optical devices, "twisted nematic" liquid crystals are used: each pixel is made of molecules with a crystalline structure, and the orientation of the molecules can be modified by the application of a voltage. The following figures illustrate the structure of a liquid crystal pixel, and are extracted from the SLM documentation `Holoeye`.

In the absence of applied voltage, the direction of the molecules follows an helical structure: there is a 90° rotation between the molecules orientation on the input face and their orientation on the output face. The figure [4.6](#) illustrates this property.

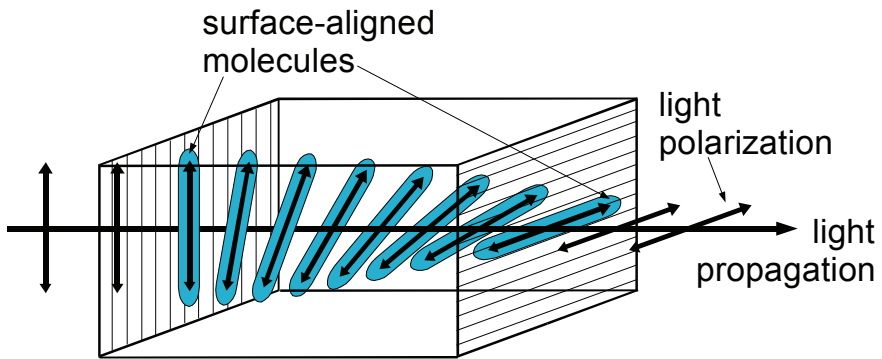


Figure 4.6: Principle of a "nematic" pixel. from Holoeye

When a voltage is applied, the orientation of the molecules is modified. The figure [4.7](#) illustrates this effect for 3 cases. In case (A) a zero voltage is applied, and the orientation of the molecules turns while remaining in the plane orthogonal to the propagation axis. In case (B), a non-zero voltage is applied, and the molecules pivot in a direction which is no more in the plane orthogonal to the propagation axis. Finally, in case (C) where a higher voltage is applied, the molecules in the center pivot in the direction of the propagation axis.

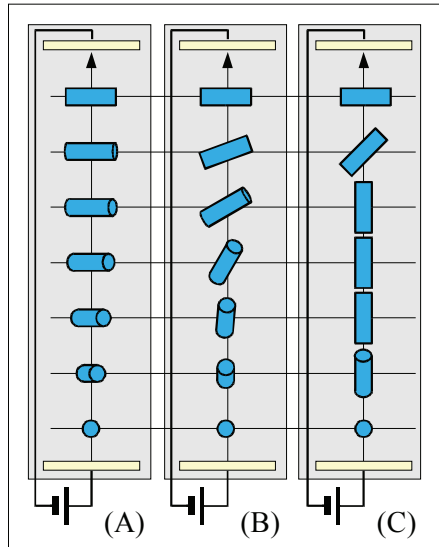


Figure 4.7: Principle of a "nematic" pixel subject to a voltage, with an increasing value from left to right from Holoeye

Birefringence of a liquid crystal

The anisotropic molecules which constitute the liquid crystals induce birefringence properties. They usually behave as a uniaxial birefringent material and one thus introduces an ordinary and an extraordinary refractive index to describe light propagation in a liquid crystal cell. The controlled orientation of the molecules allows to modify the extraordinary refractive index of the cell according to the scheme of figure [4.8](#). The propagation in the liquid crystal cell can therefore be modelled by a stack of very thin phase plates, with an orientation varying with the propagation.

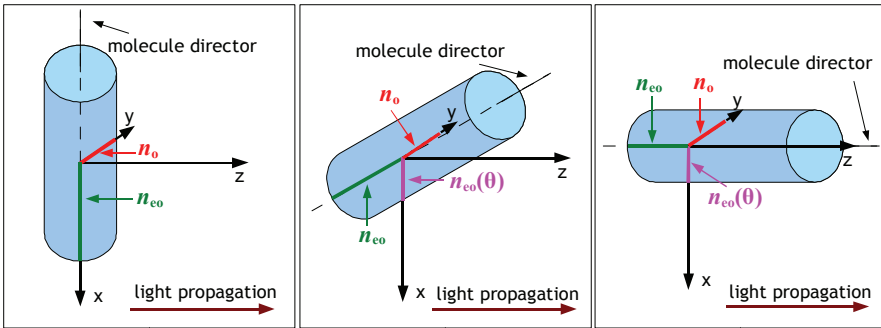


Figure 4.8: Effect of the orientation of a molecule on the effective extraordinary refractive index. n_o , n_{oe} design the ordinary and extraordinary refractive index. Depending on the orientation θ of the molecule, the effective extraordinary refractive index is noted $n_{oe}(\theta)$. from Holoeye

Annex 2 : 1D calculation of diffraction pattern of phase masks

Case of a sawtooth profile phase mask

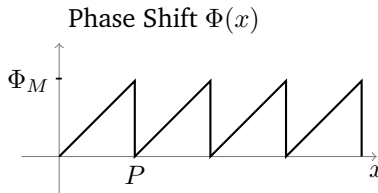


Figure 4.9: Spatial profile of a sawtooth phase grating with spacing P and modulation depth Φ_M .

The transmission $t(x) = e^{-j\Phi(x)}$ of the grating being periodic, we can express it as a convolution product with a Dirac Comb.

$$t(x) = (m \star \text{III}_P)(x)$$

Where the elementary pattern is noted $m(x)$.

The diffraction with such a grating allow to obtain in the Fourier plane, a field proportionnal to the Fourier transform of the grating, that we can express by limiting ourselves to a single direction ν as :

$$\tilde{t}(\nu) = \frac{1}{P} \cdot \tilde{m}(\nu) \cdot \text{III}_{\frac{1}{P}}(\nu)$$

We obtain as expected a series of bright spots spaced by quantity proportional to $\frac{1}{P}$. The intensity of each bright spot, i.e. of each diffraction order k is proportional to the coefficient A_k :

$$A_k = \left| \frac{1}{P} \tilde{m} \left(\frac{k}{P} \right) \right|^2$$

The expression of the Fourier transform $\tilde{m}(\nu)$ of the pattern $m(x)$ is obtained by :

$$\tilde{m}(\nu) = \int_0^P e^{-j\Phi(x)} \cdot e^{-j2\pi\nu x} dx$$

In the case of the grating of the figure [4.9](#),

$$\begin{aligned} \tilde{m}(\nu) &= \int_0^P e^{-j\frac{\Phi_M}{P} \cdot x} \cdot e^{-j2\pi\nu x} dx \\ &= \frac{-1}{j\frac{\Phi_M}{P} + j2\pi\nu} \left[e^{-j\frac{\Phi_M}{P} \cdot x} \cdot e^{-j2\pi\nu x} \right]_0^P \\ &= \frac{-1}{j\frac{\Phi_M}{P} + j2\pi\nu} \left[e^{-j\Phi_M} \cdot e^{-j2\pi\nu P} - 1 \right] \\ &= \frac{-e^{-j\frac{\Phi_M}{2}} \cdot e^{-j\pi\nu P}}{j\frac{\Phi_M}{P} + j2\pi\nu} \left[-2j \sin \left(\frac{\Phi_M}{2} + \pi\nu P \right) \right] \\ &= P e^{-j\frac{\Phi_M}{2}} \cdot e^{-j\pi\nu P} \frac{\sin \left(\frac{\Phi_M}{2} + \pi\nu P \right)}{\frac{\Phi_M}{2} + \pi\nu P} \end{aligned}$$

We can write again that Fourier transform as a dilated and shifted Cardinal Sinus :

$$\begin{aligned} \tilde{m}(\nu) &= P e^{-j\frac{\Phi_M}{2}} \cdot e^{-j\pi\nu P} \frac{\sin \left(\pi P \left(\frac{\Phi_M}{2\pi P} + \nu \right) \right)}{\frac{\Phi_M}{2} + \pi\nu P} \\ &= P e^{-j\frac{\Phi_M}{2}} \cdot e^{-j\pi\nu P} \text{sinc} \left(P \left(\frac{\Phi_M}{2\pi P} + \nu \right) \right) \end{aligned}$$

The intensities of the different diffraction orders k are then proportional to :

$$A_k = \left| \frac{\sin \left(\frac{\Phi_M}{2} + \pi k \right)}{\frac{\Phi_M}{2} + \pi k} \right|^2$$

In the case when the total phase shift brought by the phase mask is $\Phi_M = 2\pi$, then only the order -1 is non zero, as represented on figure [4.10](#). This is the analog to an reflexion echelle (or *blazed*) grating.

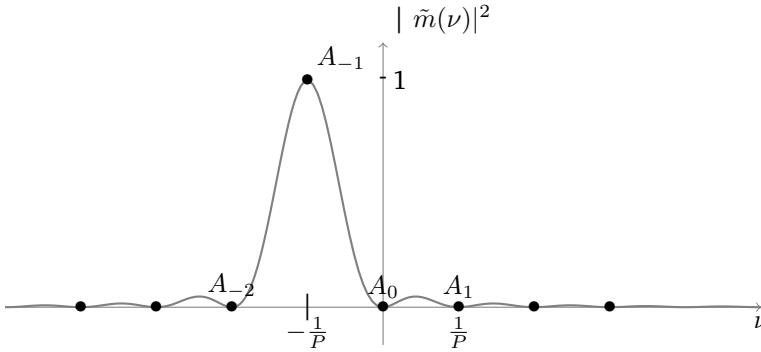


Figure 4.10: Intensity of the beam diffracted in the case of the phase mask of the figure [4.9](#), with $\Phi_M = 2\pi$

An example of a diffraction pattern obtained for $\Phi_M = \pi$ is given on figure [4.11](#).

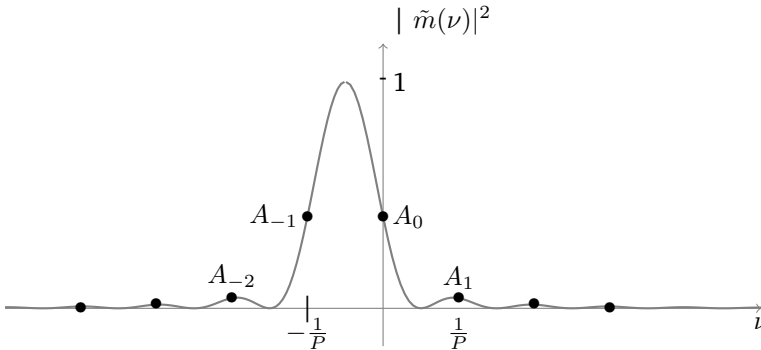


Figure 4.11: Intensity of the beam diffracted in the case of the phase mask of the figure [4.9](#), with $\Phi_M = \pi$

If the modulation depth is smaller than 2π , the measure of zeroth order :

$$A_0 = \left| \frac{\sin\left(\frac{\Phi_M}{2}\right)}{\frac{\Phi_M}{2}} \right|^2$$

and first order :

$$A_1 = \left| \frac{-\sin\left(\frac{\Phi_M}{2}\right)}{\frac{\Phi_M}{2} + \pi} \right|^2$$

Can allow to calculate the value of Φ_M .

Effect of the pixelisation

In the case of a phase mask realized by a SLM the phase shift profile is modified by pixelisation. For example, instead of a linear phase shift we observe steps as on figure [4.12](#).

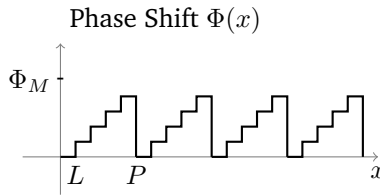


Figure 4.12: Spatial profile of a pixelated sawtooth phase grating of spacing P and modulation depth Φ_M .

The transmission $t(x) = e^{-j\Phi(x)}$ of the grating is still periodic, and we can still express it as a convolution product with a Dirac Comb.

$$t(x) = (n \star \text{III}_P)(x)$$

where the elementary pattern noted $n(x)$ is written from the previous pattern $m(x)$ as :

$$n(x) = ((m \cdot \text{III}_L) \star \text{Rect}_L)(x)$$

where the function $\text{Rect}_L(x)$ is the Gate function defined as :

$$\text{Rect}_L(x) = \begin{cases} 1 & \text{if } x \in [0, L] \\ 0 & \text{else} \end{cases}$$

The Fourier transform of the pattern $n(x)$ can then be written as :

$$\tilde{n}(\nu) = \left(\tilde{m} \star \frac{1}{L} \text{III}_{\frac{1}{L}} \right) (\nu) \cdot \widetilde{\text{Rect}_L}(\nu)$$

The diffraction pattern will then be the one of the previous continuous grating periodized and modulated by $|\widetilde{\text{Rect}_L}(\nu)|^2$. In the case of a modulation depth equal to 2π , we obtain the diffraction orders shown on figure [4.13](#)

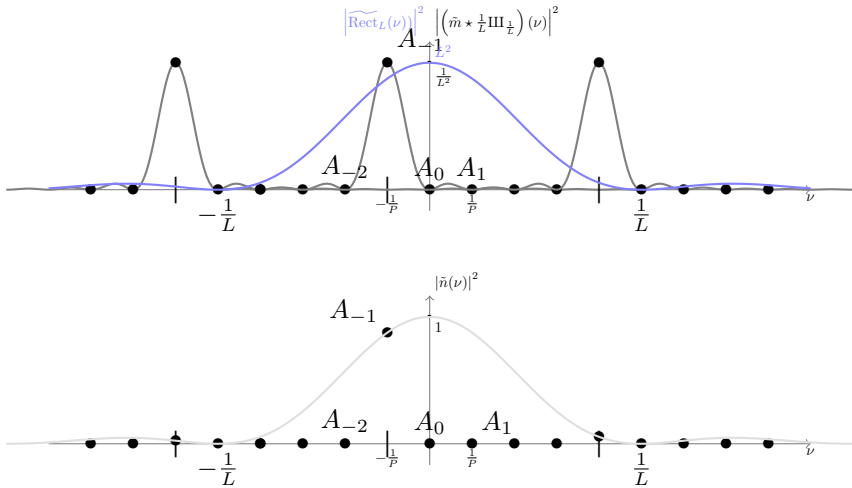


Figure 4.13: Intensity of the diffracted beam in the case of the phase mask of figure 4.12, with $\Phi_M = 2\pi$ and $P = 5L$.

Case of a rectangular profile

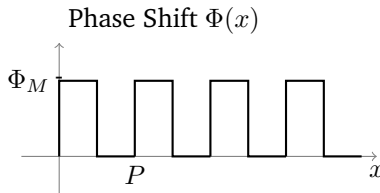


Figure 4.14: Spatial profile of a rectangular phase grating of spacing P and modulation depth Φ_M .

The Fourier transform $\tilde{m}(v)$ of the pattern $m(x)$ is still obtained with :

$$\tilde{m}(v) = \int_0^P e^{-j\Phi(x)} \cdot e^{-j2\pi vx} dx$$

In the case of the grating of figure 4.14,

$$\begin{aligned}
\tilde{m}(\nu) &= \int_0^{\frac{P}{2}} e^{-j\Phi_M} \cdot e^{-j2\pi\nu x} dx + \int_{\frac{P}{2}}^P e^{-j2\pi\nu x} dx \\
&= \frac{-1}{j2\pi\nu} e^{-j\Phi_M} [e^{-j2\pi\nu x}]_0^{\frac{P}{2}} + \frac{-1}{j2\pi\nu} [e^{-j2\pi\nu x}]_{\frac{P}{2}}^P \\
&= \frac{-1}{j2\pi\nu} e^{-j\Phi_M} [e^{-j\pi\nu P} - 1] + \frac{-1}{j2\pi\nu} [e^{-j2\pi\nu P} - e^{-j\pi\nu P}] \\
&= \frac{-1}{j2\pi\nu} e^{-j\Phi_M} [e^{-j\pi\nu P} - 1] + \frac{-1}{j2\pi\nu} e^{-j\pi\nu P} [e^{-j\pi\nu P} - 1] \\
&= \frac{-[e^{-j\pi\nu P} - 1]}{j2\pi\nu} \cdot [e^{-j\Phi_M} + e^{-j\pi\nu P}] \\
&= \frac{-[e^{-j\pi\nu P} - 1]}{j2\pi\nu} \cdot e^{-j\Phi_M} [1 + e^{-j\pi\nu P + j\Phi_M}] \\
&= \frac{-[e^{-j\pi\nu P} - 1]}{j2\pi\nu} \cdot e^{-j\Phi_M} \cdot e^{-j\pi\nu \frac{P}{2} + j\frac{\Phi_M}{2}} \left[e^{-(-j\pi\nu \frac{P}{2} + j\frac{\Phi_M}{2})} + e^{-j\pi\nu \frac{P}{2} + j\frac{\Phi_M}{2}} \right] \\
&= \frac{-[e^{-j\pi\nu P} - 1]}{j2\pi\nu} \cdot e^{-j\Phi_M} \cdot e^{-j\pi\nu \frac{P}{2} + j\frac{\Phi_M}{2}} \left[2 \cos \left(-\pi\nu \frac{P}{2} + \frac{\Phi_M}{2} \right) \right]
\end{aligned}$$

The intensities of diffraction orders are proportional to :

$$A_k = \left| \frac{1}{P} \tilde{m} \left(\frac{k}{P} \right) \right|^2$$

The first term in the expression of $\tilde{m}(\nu)$ is zero for the even values of k , except 0. For the odd terms, the modulus of this first term is equal to $\frac{P}{k\pi}$.

$$A_k = \left| \frac{1}{k\pi} \cdot \left[2 \cos \left(-\frac{k\pi}{2} + \frac{\Phi_M}{2} \right) \right] \right|^2$$

$$A_k = \frac{4}{k^2\pi^2} \cdot \left(\sin \left(\frac{\Phi_M}{2} \right) \right)^2$$

$$A_k = \frac{2}{k^2\pi^2} \cdot (1 - \cos(\Phi_M))$$

To calculate order 0, one has to express the first term of the expression of $\tilde{m}(\nu)$:

$$\begin{aligned}
\frac{-[e^{-j\pi\nu P} - 1]}{j2\pi\nu} &= e^{-j\frac{\pi\nu P}{2}} \cdot \frac{-[e^{-j\frac{\pi\nu P}{2}} - e^{+j\frac{\pi\nu P}{2}}]}{j2\pi\nu} \\
&= e^{-j\frac{\pi\nu P}{2}} \cdot \frac{-[-2j \sin \left(\frac{\pi\nu P}{2} \right)]}{j2\pi\nu} = e^{-j\frac{\pi\nu P}{2}} \cdot \frac{P}{2} \cdot \frac{[\sin \left(\frac{\pi\nu P}{2} \right)]}{\frac{\pi\nu P}{2}}
\end{aligned}$$

The modulus of this first term is then equal to $\frac{P}{2}$ for $\nu = 0$.

The coefficient corresponding to order 0 expresses then as :

$$A_0 = \left(\cos \left(\frac{\Phi_M}{2} \right) \right)^2 = \frac{1}{2} (1 + \cos(\Phi_M))$$

For a maximum phase shift of $\Phi_M = 2\pi$, only zeroth order is non zero.

On the contrary, for a phase shift depth of $\Phi_M = \pi$, the zeroth order is zero.

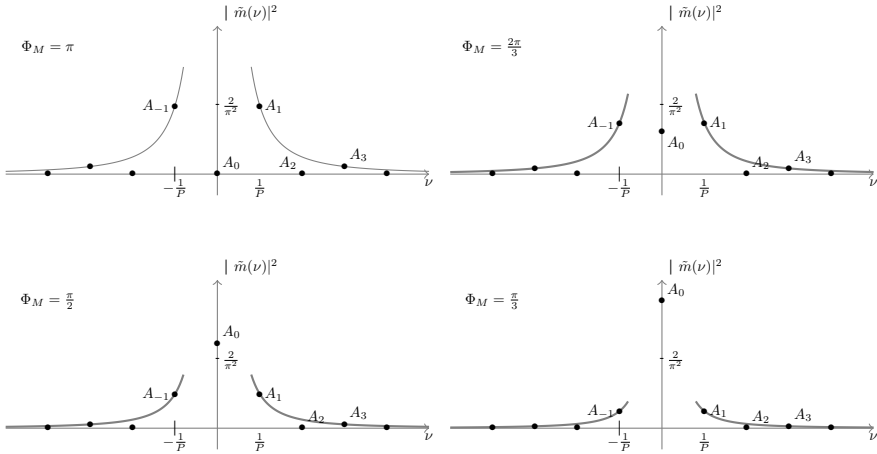


Figure 4.15: Intensity of the diffracted beam in the case a phase mask of figure 4.14, for different values of the modulation depth Φ_M