Lab work in photonics.

Polarization.

Rooms	**P1**	**P2**	**P3**	**P4**
	S1.15	S1.13	S1.19	S1.13

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Engineer - 2nd year - S7 -Palaiseau
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Polarization : "Well begun is half-done [1]"

The purpose of polarization labs is to illustrate recurring experimental situations where you will be brought to understand the effect of polarization on the propagation of a light wave. They use some of the concepts covered during the first year course and it is important to read it again before starting the labs.

At the end of the session, you will able to:

- produce a given polarization state,
- analyze a given polarization state, using several methods,
- measure a linear birefringence, using several methods,
- characterize a medium having a circular birefringence,
- make an amplitude modulation with electro-optic or liquid cristal materials .

The following few basic questions should help you prepare the polarization labs.

P1 What is the effect of a polarizer on light?

P2 What is meant by saying that a material is "birefringent"?

P3 What is the operating principle of a wave plate? What is the definition of a neutral axis?

P4 What is the phase shift introduced by a $\lambda/4$ plate (quarter-wave plate or QWP)? Same question with a $\lambda/2$ plate (half-wave plate or HWP).

P5 What is the effect of these plates on an incident linear polarization state?

P6 Do the properties of a wave plate depend on wavelength? If yes, how so?

P7 What is the "ellipticity" of polarized light?

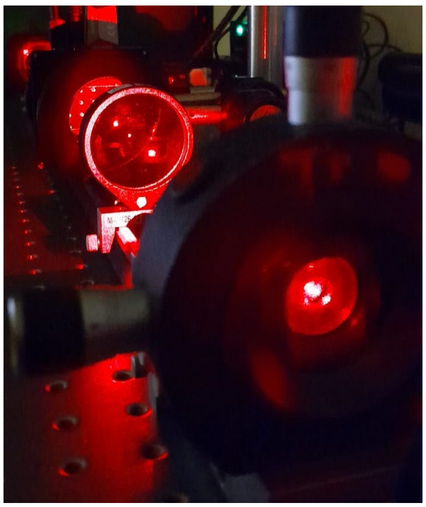
P8 What is the polarization exiting a wave plate when it is illuminated by a linearly polarized light at 45° from the neutral axes of the plate?

[1] Mary Poppins, 1964

P9 How are "transverse electric" (TE or S) and "transverse magnetic" (TM or P) polarizations defined? What is Brewster's angle?

Lab 1

Spatial Light modulator



At the end of this session you will be able to:

- set up an optical bench in imaging and diffraction configuration,
- describe the operating principle of an imaging SLM (amplitude modulation)
- measure the characteristics of an SLM (pixel size, polarization states obtained at output)
- describe the operating principle of an SLM used in diffractive devices (phase modulation)

Contents

1 Introduction

⇝ Place a polarizer in front of the screens available in the room (the 2 computer screens, your smartphone, etc.) and rotate it.

Q1 Is the light from a liquid crystal screen polarized?

The polarization of light is the characteristic that a liquid crystal modifies. A spatial light modulator consists of a matrix of liquid crystals. Each pixel is controlled by an electrical voltage, so each pixel can independently modify the polarization of the light wave passing through it.

Depending on the device into which the modulator is inserted, this polarization control can be used for different applications.

In this session, you'll use a spatial light modulator (SLM) for two types of application: amplitude modulation for imaging, and phase modulation for diffractive masks (holograms, DOE Diffractive optical elements).

2 Introduction to the experiment bench

The SLM used in this session (`Holoeye - LC2012`, an extract of whose documentation is shown in figure 1.1) is controlled via a VGA signal by a computer interface developed under `Matlab`.

LC 2012 Spatial Light Modulator – Microdisplay Features	
Display Type:	Transmissive LC
Resolution:	1024 x 768
Pixel Pitch:	36 µm
Fill Factor:	58 %
Active Area	36.9 x 27.6 mm (1.8" Diagonal)
Addressing	8 Bit (256 Grey Levels)
Signal Formats	HDMI – XGA Resolution
Input Frame Rate	60 Hz

Figure 1.1 – Extract from SLM documentation `Holoeye - LC2012`

This interface is used to display images on the SLM and to retrieve the signal from the observation camera. It transforms the grayscale-coded image signal into a control voltage for the SLM, according to a programmable transfer function (similar to contrast or brightness settings on monitors). In our case, this transfer function has been optimized for the various experiments required in the practical work you're about to perform. The gray levels of the image (whose link with the applied voltage is therefore masked by the software) will therefore serve as a reference throughout the practical work.

The SLM operates in transmission mode, and is inserted on the bench between two polarization control elements :

- an input polarizer
- and an analyzer for selecting an output polarization direction.

⇝ Check the orientation of these two polarizers using the Brewster phenomenon (e.g. with light from hallway ceiling lights or at an air-glass interface).

The bench is shown in the photo in figure 1.2. A first lens (L_1) is used to collimate the beam, a second (L_2) to image the SLM on a screen or on the plane of the observation camera.

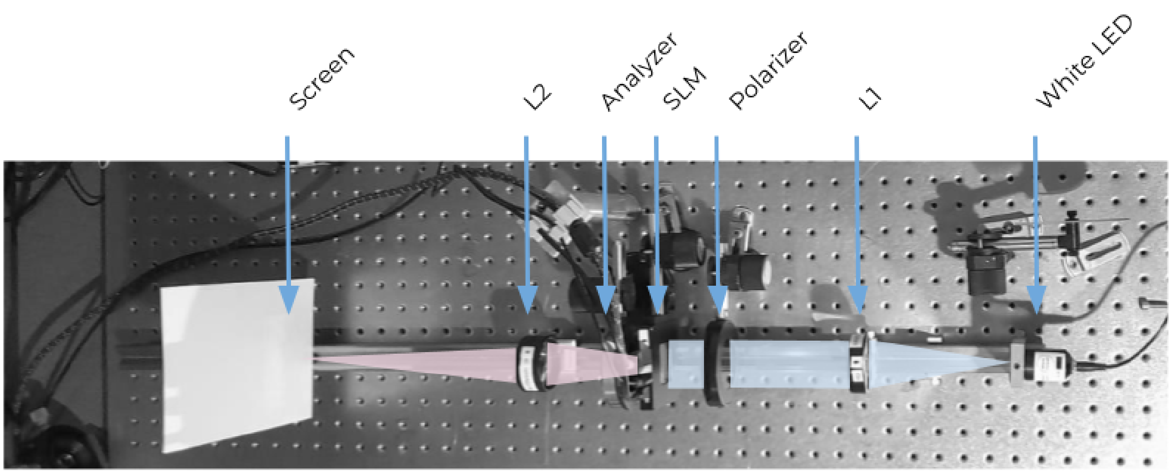


Figure 1.2 – The different elements of the setup. Imaging configuration.

3 First white-light observations

The diagram in figure 1.3 describes the orientations of the various bench elements in the imaging configuration used in this section.

⇝ Switch on the white source, set the position of the L_1 objective by autocollimation to illuminate the SLM with a parallel beam.

⇝ Orient the polarizer so that the input polarization is rectilinear and horizontal (the axes of the polarizer correspond to the 0 graduation), (Let $\theta_P = 0$ on the figure 1.3).

⇝ Place the analyzer axis in the horizontal position ($\theta_A = 0$ on figure 1.3).

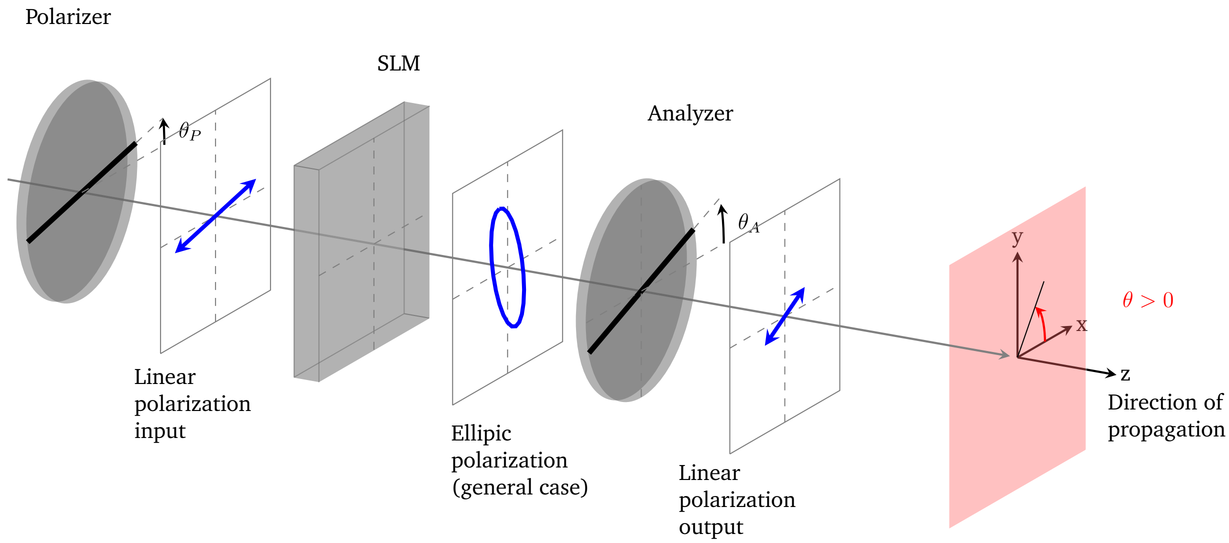


Figure 1.3 – Diagram of assembly in imaging configuration. Orientation of the various elements.

⇝ Power up the SLM. Launch the interface software for the SLM and camera. To do this, launch `matlab` and type »`SLM` at the command line.

⇝ Apply a zero gray level (NG=0) to one half of the SLM surface, and a maximum gray level (NG=255) to the other half, using the `Coupe Verticale` menu.

We'd like to observe the image of the SLM on a screen placed at the end of the bench, using a Clairault doublet lens.

Q2 Which conjugation gives a transverse magnification $g_y = 1$? $g_y = 2$? (Draw a diagram).

⇝ Choose one of the two objectives at your disposal, adjust its position (and direction of use) and that of the screen to obtain a sharp image downstream of the analyzer to make the sharp image of the SLM on the screen with a magnification greater than 1.

Q3 Do pixels controlled at gray level 0 (NG=0, i.e. black pixels in the control image) appear black on the screen ? What can you deduce about the behavior of SLM pixels for NG=0?

⇝ Rotate the analyzer by 90°.

Q4 Explain the changes made to the image.

⇝ Load an image into the software and compare again the images obtained for two orthogonal analyzer positions. To do this, use the menu `Load Image`.

⇝ Load an image of the type "saw-tooth grating" and observe the images obtained for different analyzer positions. To do this, use the `Ajout d'un réseau blazé` menu.

Q5 Explain why the resulting image is colored.

⇝ Place a green or red filter in front of the white source and observe the image obtained for different control images, different analyzer positions, without the analyzer, etc.

Q6 Explain the advantages of keeping a filter for amplitude modulation measurements.

4 Use of SLM for imaging (amplitude modulation)

In this section, we want to measure the illuminance obtained in the image plane as a function of the gray level of the control image. To do this, we'll place a camera in the image plane.

4.1 Bench and camera settings

Q7 What is the sensor size of the camera at your disposal? That of the SLM? What conjugation must be performed to obtain a transverse magnification g_y allowing you to see the entire SLM image on the camera?

⇝ Choose the lens, and adjust its position and that of the camera approximately so that the image of the SLM is in the plane of the camera sensor at the appropriate magnification.

⇝ Initialize the camera. To do this, select the `Initialisation Caméra` menu and set the exposure time.

⇝ Re-apply zero gray level (NG=0) to one half of the SLM surface, and maximum gray level (NG=255) to the other half, using the `Coupe Verticale` menu.

⇝ Set polarizer and analyzer orientation to $\theta_P = 0^\circ$ and $\theta_A = 90^\circ$.

⇝ Adjust the position of the lens and/or camera to obtain a sharp image on the camera sensor.

4.2 Analysis of output polarization state

⇝ Select the menu `Analyse de la polarisation`.

⇝ Plot the curves of the average signal received by the camera as a function of the θ_A orientation of the analyzer axis, for control gray levels NG=0 and NG=255.

Q8 Explain the shape of these two curves. Is the polarization at the SLM output straight?

Q9 Give the expression for the ellipticity of the polarization as a function of the curve parameters. Determine the orientation and ellipticity of the polarization at the SLM output for a control gray level at NG=0 and NG=255 from your two curves. Compare the values you find with those determined directly by the software (by recording the minimum and maximum signal values).

4.3 Use in amplitude modulation

Q10 What is the position of the analyzer axis for maximum contrast? The position to obtain the brightest image, for NG=0? For NG=255?

⇝ Place the analyzer in the orientation ensuring maximum contrast and plot the curve giving the camera signal as a function of the control gray level applied to the pixel.

Q11 Is this curve linear? What kind of curve would be obtained if the SLM only modified the polarization orientation as a function of the control gray level?

⇝ (Repeat the previous measurement, replacing the filter (green or red).)

Q12 Compare the curves obtained with those given by the manufacturer. Documentation is available in the room.

5 Replace the source with a laser source

The source used in this and the next section is a `HeNe` laser. In order to illuminate the entire surface of the SLM with the laser beam, beam shaping steps are required on the bench (see figure 1.6) :

- a first lens is used to widen the beam at the laser output,
- a cleaning device evens out the distribution of illumination in the beam,
- finally, the L_1 objective is used to collimate the beam in order to illuminate the SLM as a parallel beam (as in the imaging configuration).

⇝ Align and adjust the position of these elements. We suggest the following protocol :

- remove the white source and filter from the bench,
- switch on the laser `HeNe` and check that the beam is centered on the first mirror M_1.
- position (and clamp) the second mirror M_2 in the axis of the bench,

- place the (fixed) adjustment hole at the end of the bench and adjust mirror M_1 so that the beam is centered,
- move the adjustment hole away from the bench and adjust the orientation of mirror M_2 so that the beam is centered,
- place the microscope objective on the bench,
- make sure the beam is present at the exit of the microscope objective, adjust the orientation of mirror M_2 and possibly M_1 if necessary,
- place the cleaning hole in front of the microscope objective,
- adjust the transverse positions of the objective and the longitudinal position of the hole in order to obtain a uniform distribution of the beam intensity over the entire pupil of the objective L_1,
- finally, autocollimate the position of lens L_1 to illuminate the SLM with a parallel beam.

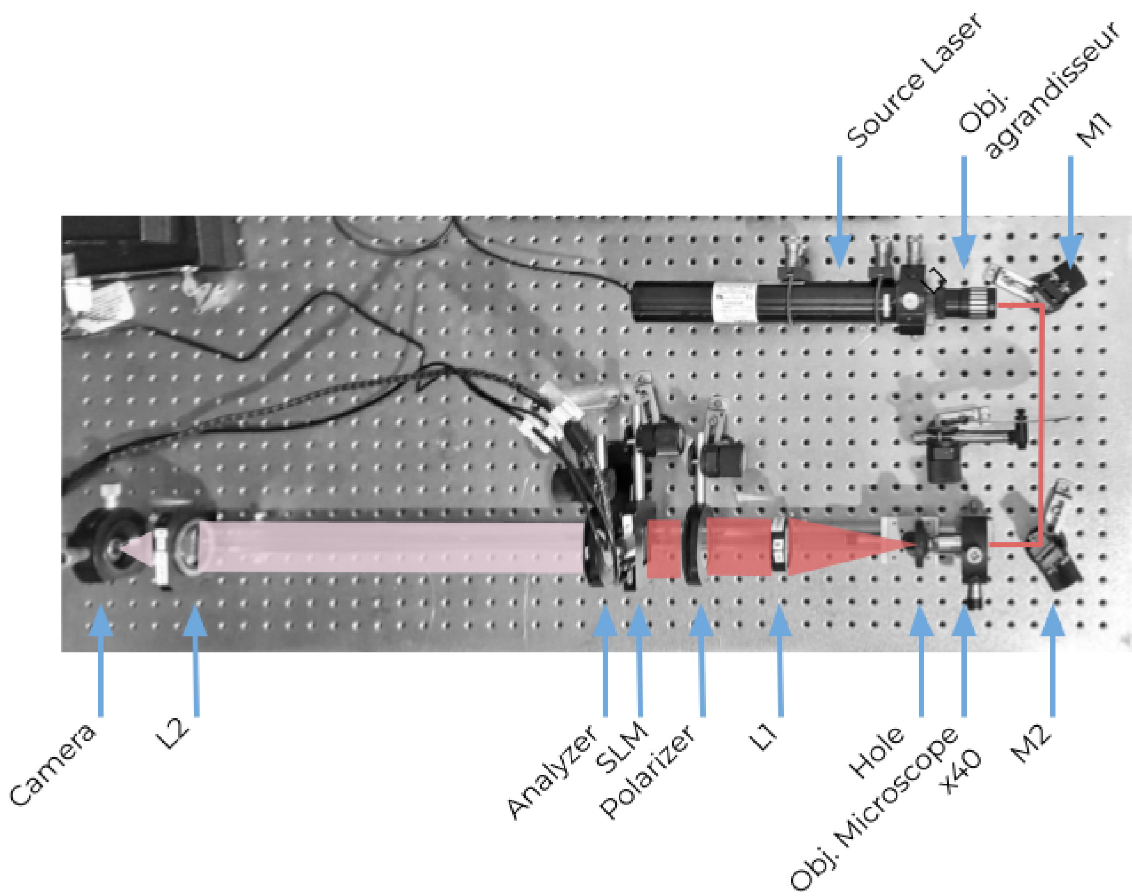


Figure 1.4 – The different elements of the set-up. Laser source imaging setup

⇝ Observe the image obtained in the camera plane using the L_2 lens with focal length $f' = 120\,\mathrm{mm}$, adjust the beam intensity (using the adjustable density) and/or the camera exposure time if necessary.

Q13 Comment and explain the differences between the image obtained with the laser source and that obtained with the LED source.

Q14 Why aren't laser sources used in imaging devices?

Q15 Why is a laser source used to illuminate diffractive elements?

6 Using the SLM for phase modulation

The diagram in figure 1.5 describes the orientations of the various bench elements in the phase modulation configuration used in this section.

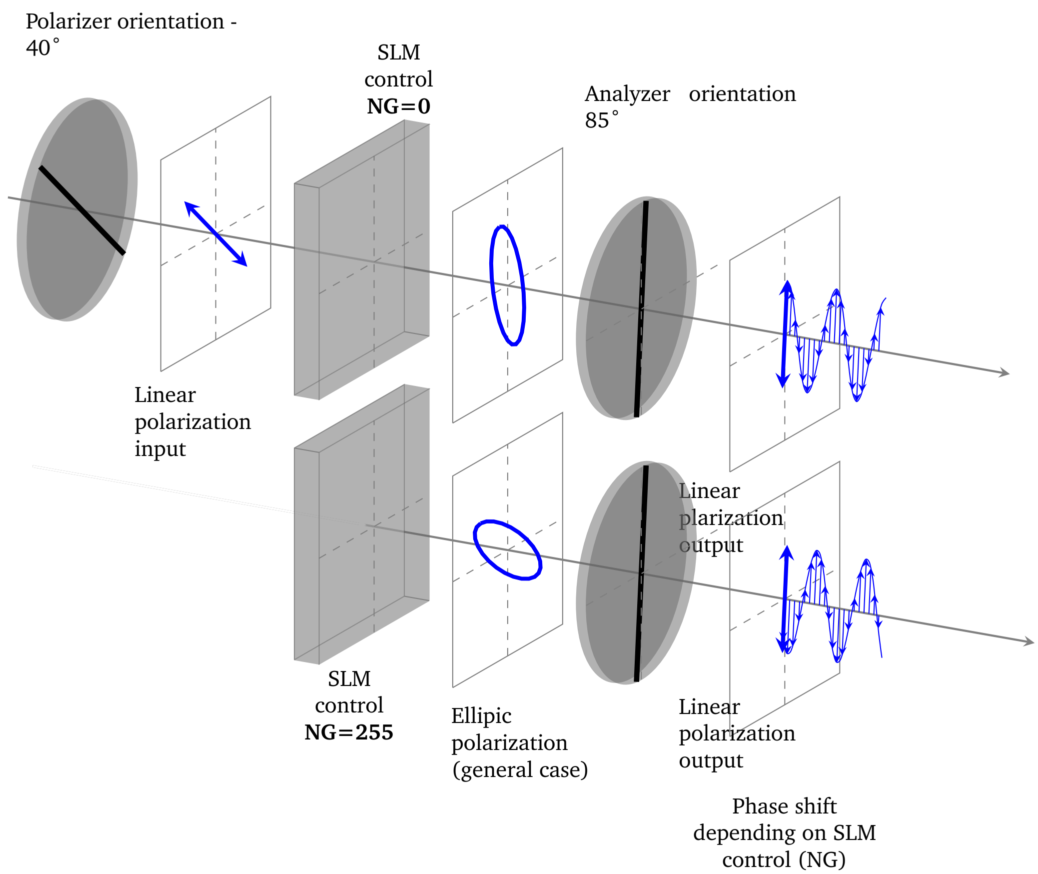


Figure 1.5 – Schematic diagram of phase modulation configuration. Orientation of the various elements and illustration of the phase modulation principle.

6.1 Setting polarizer and analyzer orientation

⇝ Re-apply zero gray level (NG=0) to one half of the SLM surface, and maximum gray level (NG=255) to the other half, using the `Coupe Verticale` menu.

⇝ Set the polarizer axis to $\theta_P = -40^\circ$. Set the analyzer axis to $\theta_A = 85^\circ$. This configuration is indicated in the documentation for phase modulation without amplitude modulation.

Q16 Are there any sensors that are sensitive to the phase of a light wave? Why is there little difference in amplitude between the two areas of the camera image?

6.2 Observations in the Fourier plane.

Sculpting the phase of a wave allows you to control the distribution of illumination in the Fourier plane with great precision and resolution.

⇝ Apply uniform control to the SLM.

⇝ Move the lens L_2 so that the camera sensor is in its Fourier plane (see figure 1.6).

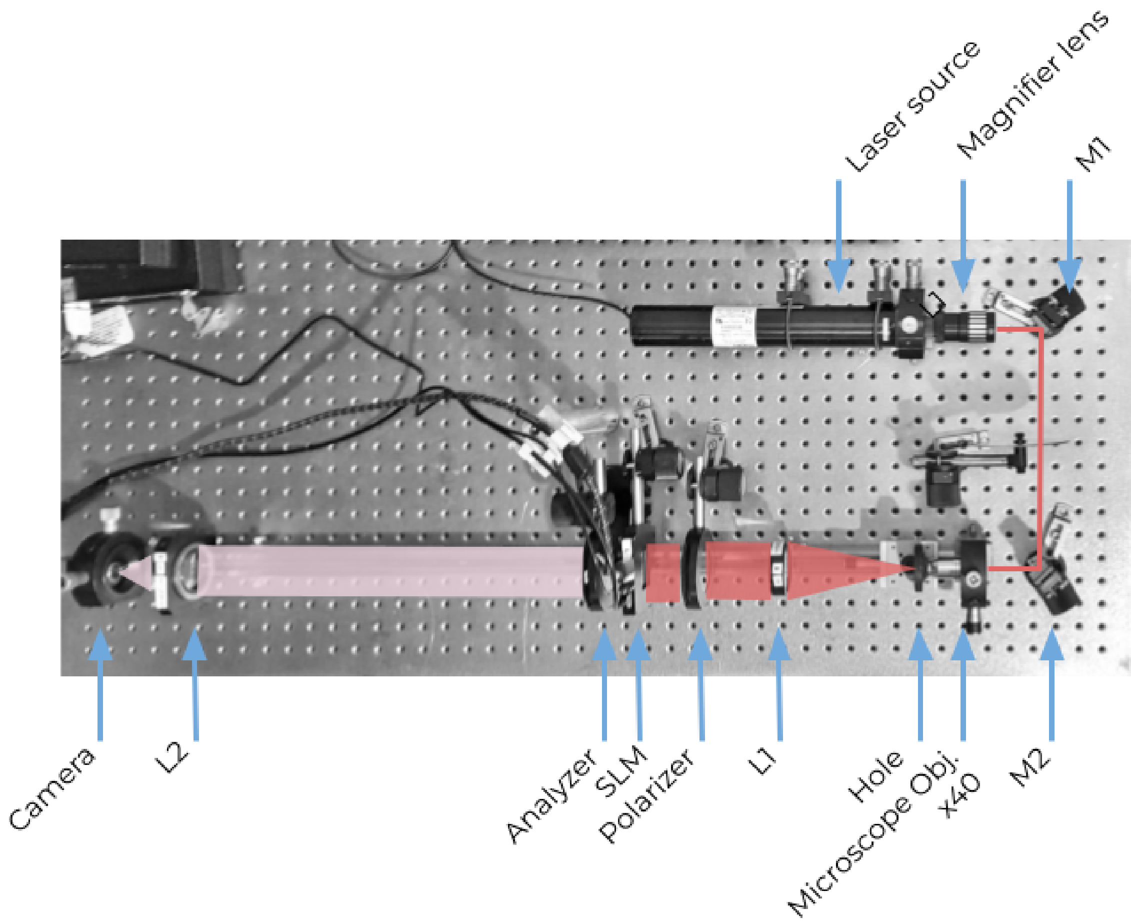


Figure 1.6 – The different elements of the assembly. Phase modulation configuration (only the L_2 lens is moved).

Q17 Why do several diffraction peaks appear in the horizontal and vertical directions on the camera image ?

⇝ Use the `Profil de l'image` tool to measure the spacing between these peaks in number of camera pixels. Make sure the camera is not saturated; use variable density if necessary.

Q18 Give the value of the spacing between peaks in mm. Derive an estimate of the SLM's pixel size and compare your estimate with the manufacturer's data.

6.3 Gratings and defocusing

⇝ Load the image of a linear grating onto the SLM using the menu `Add a blazed grating`. Set the step size to a sufficiently small value (< 50 pixels).

Q19 How is the diffraction pattern obtained on the camera modified?

⇝ Use the `Profil de l'image` tool to measure the relative amplitudes of the main peaks.

Q20 Can the SLM produce a phase shift of $2pi$? de π? (see appendix)

Q21 Estimate the modulation depth from the relative amplitudes of the main peaks and compare with the data in the documentation.

⇝ Load the image of a radial variation `Lentille de Fresnel` and check that the Fourier plane has changed position, by moving the lens L_2 or the camera plane.

6.4 Digital hologram

⇝ Use the `Profil de l'image` command to load one of the "Mystery" images.

Q22 Observe the diffraction pattern in the Fourier plane. Comment on it!

⇝ Remove the analyzer from the set-up.

Q23 Comment on the differences observed in the Fourier plane image.

7 Conclusion

Q24 Summarize the differences in experimental conditions for modulating amplitude or phase using the SLM:

- coherent source or not?
- imaging at finite distance or ... ?
- relationship between SLM command and desired image?

Annex 1: Liquid Crystals Physics

Liquid crystals present a structure which associates both the properties of a crystal and those of a liquid. These properties were discovered at the end of the XIXth century and the first screens using liquid crystals pixels date from the 1970s.

Structure of a nematic pixel

In optical devices, "twisted nematic" liquid crystals are used: each pixel is made of molecules with a crystalline structure, and the orientation of the molecules can be modified by the application of a voltage. The following figures illustrate the structure of a liquid crystal pixel, and are extracted from the SLM documentation `Holoeye`.

In the absence of applied voltage, the direction of the molecules follows an helical structure: there is a 90°rotation between the molecules orientation on the input face and their orientation on the output face. The figure 1.7 illustrates this property.

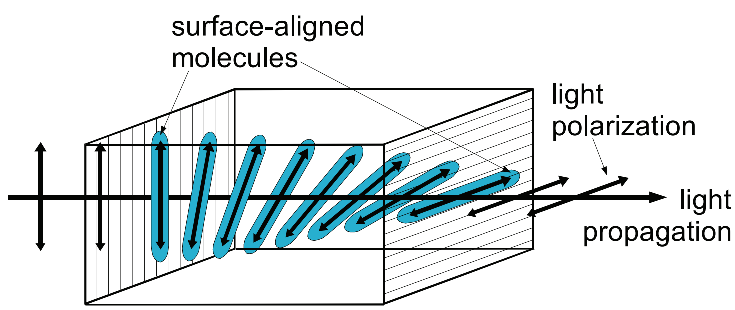


Figure 1.7 – Principle of a "nematic" pixel. from Holoeye

When a voltage is applied, the orientation of the molecules is modified. The figure 1.8 illustrates this effect for 3 cases. In case (A) a zero voltage is applied, and the orientation of the molecules turns while remaining in the plane orthogonal to the propagation axis. In case (B), a non-zero voltage is applied, and the molecules pivot in a direction which is no more in the plane orthogonal to the propagation axis. Finally, in case (C) where a higher voltage is applied, the molecules in the center pivot in the direction of the propagation axis.

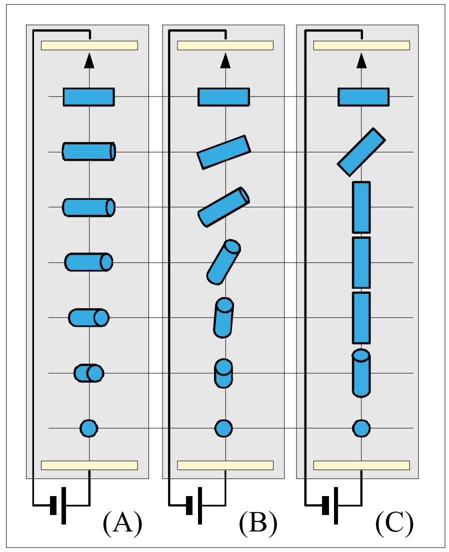


Figure 1.8 – Principle of a "nematic" pixel subject to a voltage, with an increasing value from left to right
from Holoeye

Birefringence of a liquid crystal

The anisotropic molecules which constitute the liquid crystals induce birefringence properties. They usually behave as a uniaxial birefringent material and one thus introduces an ordinary and an extraordinary refractive index to describe light propagation in a liquid crystal cell. The controlled orientation of the molecules allows to modify the extraordinary refractive index of the cell according to the scheme of figure 1.9. The propagation in the liquid crystal cell can therefore be modelled by a stack of very thin phase plates, with an orientation varying with the propagation.

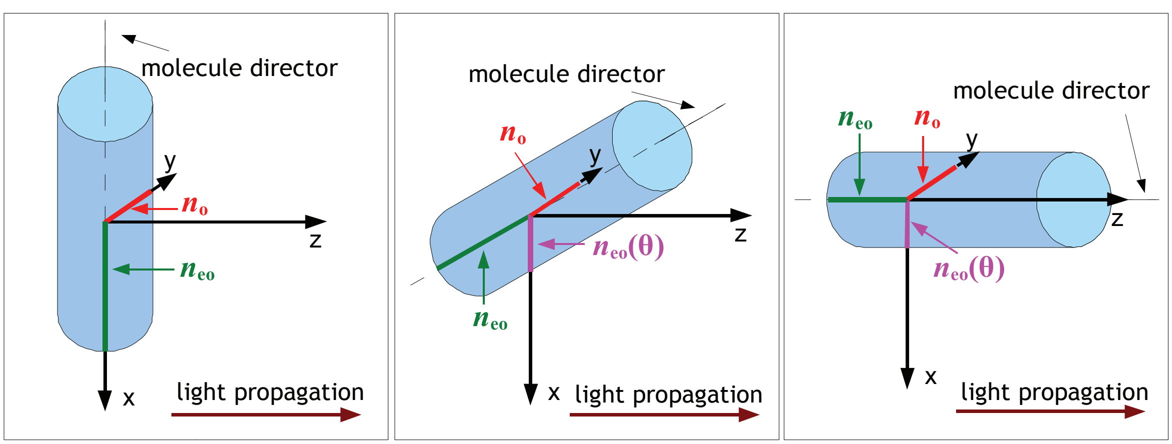


Figure 1.9 – Effect of the orientation of a molecule on the effective extraordinary refractive index. n_o, n_{oe} design the ordinary and extraordinary refractive index. Depending on the orientation θ of the molecule, the effective extraordinary refractive index is noted $n_{oe}(\theta)$. from Holoeye

Annex 2: 1D calculation of diffraction pattern of phase masks

Case of a sawtooth profile phase mask

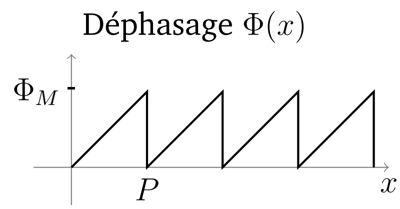


Figure 1.10 – Spatial profile of a sawtooth phase grating with spacing P and modulation depth Φ_M.

The transmission $t(x) = \mathrm{e}^{-j\Phi(x)}$ of the grating being periodic, we can express it as a convolution product with a Dirac Comb.

$$t(x) = (m \star \text{Ш}_p)(x)$$

Where the elementary pattern is noted $m(x)$.

The diffraction with such a grating allow to obtain in the Fourier plane, a field proportionnal to the Fourier transform of the grating, that we can express by limiting ourselves to a single direction ν as :

$$\tilde{t}(\nu) = \frac{1}{P} \cdot \tilde{m}(\nu) \cdot \text{Ш}_{\frac{1}{P}}(\nu)$$

We obtain as expected a series of bright spots spaced by quantity proportional to $\frac{1}{P}$. The intensity of each bright spot, i.e. of each diffraction order k is proportional to the coefficient A_k :

$$A_k = \left| \frac{1}{P} \tilde{m}\left(\frac{k}{P}\right) \right|^2$$

The expression of the Fourier transform $\tilde{m}(\nu)$ of the pattern $m(x)$ is obtained by :

$$\tilde{m}(\nu) = \int_0^P \mathrm{e}^{-j\Phi(x)} \cdot \mathrm{e}^{-j2\pi\nu x} dx$$

In the case of the grating of the figure 1.10,

$$
\begin{aligned}
\tilde{m}(\nu) &= = \int_0^P \mathrm{e}^{-j\frac{\Phi_M}{P}\cdot x} \cdot \mathrm{e}^{-j2\pi\nu x} dx \\
&= \frac{-1}{j\frac{\Phi_M}{P} + j2\pi\nu} \left[\mathrm{e}^{-j\frac{\Phi_M}{P}\cdot x} \cdot \mathrm{e}^{-j2\pi\nu x}\right]_0^P \\
&= \frac{-1}{j\frac{\Phi_M}{P} + j2\pi\nu} \left[\mathrm{e}^{-j\Phi_M} \cdot \mathrm{e}^{-j2\pi\nu P} - 1\right] \\
&= \frac{-\mathrm{e}^{-j\frac{\Phi_M}{2}} \cdot \mathrm{e}^{-j\pi\nu P}}{\frac{j\Phi_M}{P} + j2\pi\nu} \left[-2j\sin\left(\frac{\Phi_M}{2} + \pi\nu P\right)\right] \\
&= P\mathrm{e}^{-j\frac{\Phi_M}{2}} \cdot \mathrm{e}^{-j\pi\nu P} \frac{\sin\left(\frac{\Phi_M}{2} + \pi\nu P\right)}{\frac{\Phi_M}{2} + \pi\nu P}
\end{aligned}
$$

We can write again that Fourier transform as a dilated and shifted Cardinal Sinus :

$$
\begin{aligned}
\tilde{m}(\nu) &= P\mathrm{e}^{-j\frac{\Phi_M}{2}} \cdot \mathrm{e}^{-j\pi\nu P} \frac{\sin\left(\pi P(\frac{\Phi_M}{2\pi P} + \nu)\right)}{\frac{\Phi_M}{2} + \pi\nu P} \\
&= P\mathrm{e}^{-j\frac{\Phi_M}{2}} \cdot \mathrm{e}^{-j\pi\nu P} \mathrm{sinc}\left(P\left(\frac{\Phi_M}{2\pi P} + \nu\right)\right)
\end{aligned}
$$

The intensities of the different diffraction orders k are then proportional to :

$$
A_k = \left|\frac{\sin\left(\frac{\Phi_M}{2} + \pi k\right)}{\frac{\Phi_M}{2} + \pi k}\right|^2
$$

In the case when the total phase shift brought by the phase mask is $\Phi_M = 2\pi$, then only the order -1 is non zero, as represented on figure 1.11. This is the analog to an reflexion echelle (or *blazed*) grating.

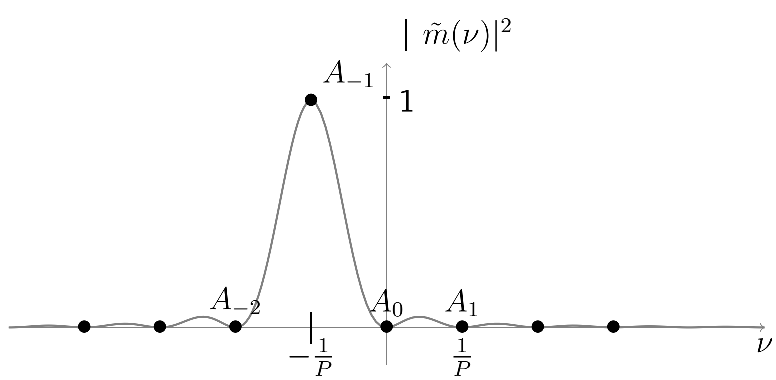


Figure 1.11 – Intensity of the beam diffracted in the case of the phase mask of the figure 1.10, with $\Phi_M = 2\pi$

An example of a diffraction pattern obtained for $\Phi_M = \pi$ is given on figure 1.12.

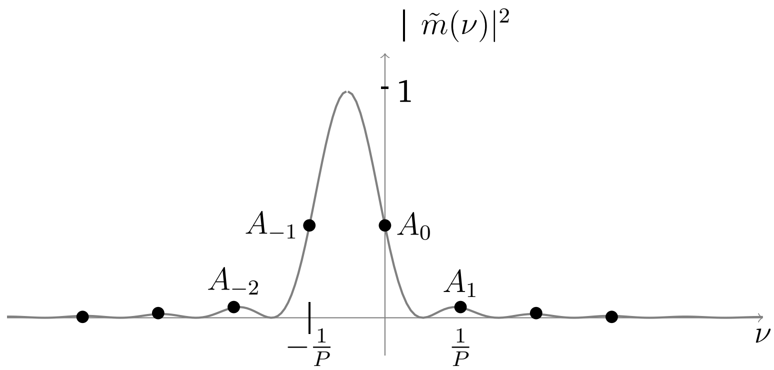


Figure 1.12 – Intensity of the beam diffracted in the case of the phase mask of the figure 1.10, with $\Phi_M = \pi$

If the modulation depth is smaller than 2π, the measure of zeroth order :

$$A_0 = \left| \frac{\sin\left(\frac{\Phi_M}{2}\right)}{\frac{\Phi_M}{2}} \right|^2$$

and first order :

$$A_1 = \left| \frac{-\sin\left(\frac{\Phi_M}{2}\right)}{\frac{\Phi_M}{2} + \pi} \right|^2$$

Can allow to calculate the value of Φ_M.

Lab 2

Birefringence measurements

The objective of this session is to determine as accurately as possible the birefringence of some crystalline plates (that is to say the path difference (OPD) introduced between the two neutral axes) using different methods.

At the end of this session, you will be able to :

- align and optimize a bench for measuring linear birefringence using the spline spectrum method
- calibrate a Babinet compensator
- determine the orientation of the neutral axes of a birefringent plate
- determine the value of the linear birefringence of blades, for which you will be able :
  - to interpret the dark bangs of the spline spectrum,
  - to interpret the displacement of bangs in a Babinet compensator,
  - to model the polarization of light at the various levels of the measurement benches,
  - to compare results obtained with two different methods,
  - to process the data and estimate the measurement uncertainty reliably.

Answering the question **P1** in advance will help you to be more effective in the session.

Contents

Depending on the samples, some methods are applicable or not, more or less accurate, complementary... In any case, it is essential to exploit all the results on site to immediately detect any incompatibilities between the measurements, and **make all the necessary verifications on site**. We will try, for two methods, to estimate the uncertainties and to minimize them.

These samples are really fragile and expensive (about 300 euros P.U.). Therefore, you must handle them with care and put them back in their boxes after use.

Methods used:

- Observation of a channeled spectrum with a USB grating spectrometer
- Measurement of the OPD with a Babinet compensator

During the lab session, you will summarize all the obtained results, for each plate, in a table. You will also draw $\delta(\lambda)$, the OPD as a function of λ for all the measurements and you will verify the obtained trend and the consistency of your results.

1 Preparation

P1 What kind of polarization state exits a birefringent plate when the incident polarization is linear and oriented at $45°$ from the neutral axes of the plate? Precise the orientation of the exiting polarization state. Draw a scheme showing the incident polarization, the neutral axes of the plate, and the exiting polarization state. We keep aside the handedness of the exiting polarization.

P2 Give the relationship between the ellipticity ε of the exiting polarization and the phase shift φ introduced by the plate.

P3 How must be orientated the optical axis of the crystal with respect to the propagation axis in order to observe the effect of their birefringence ?

2 Study of the dark fringes in the white light spectrum

$\rightsquigarrow$ Carefully align the setup. That means carefully align all the optical components on the optical bench, fix the lenses at the correct position with the auto-collimation method, in order to shine collimated light on the sample. This setup is the same as the one used during the tutorial Polarization 1 to study the quartz rotatory power.

$\rightsquigarrow$ Orient the polarizer at $45°$ with respect to the vertical axis, and cross precisely the analyzer with it.

$\rightsquigarrow$ Place the plate under study on the bench and orient its axes at $45°$ to those of the polarizer and the analyzer.

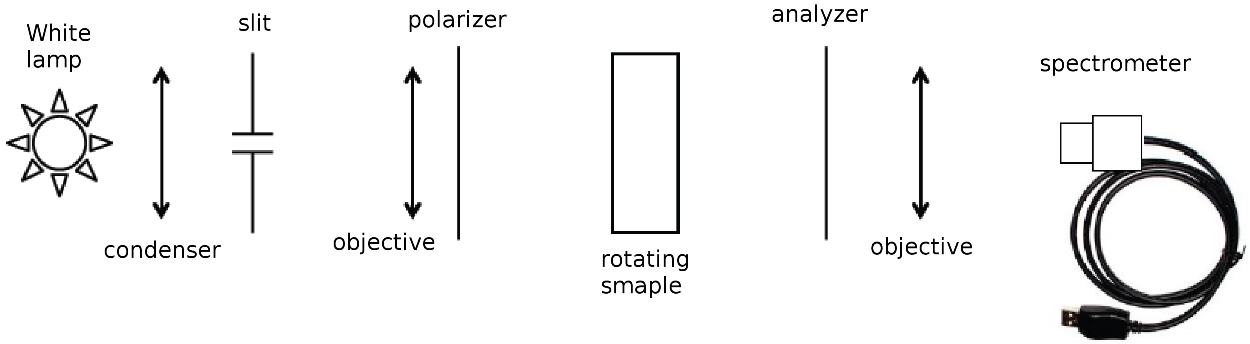


Figure 2.1 – Experimental setup

Q1 Explain clearly and simply the presence of dark fringes in the spectrum between parallel polarizers and between crossed polarizers.

Q2 What is the effect on the spectrum of a sample rotation around the optical axis of the set-up?

The positions of the dark fringes **between polarizer and analyzer either crossed or parallel** can be traced back to the value of the optical path difference for specific wavelength values.

The contrast of the interference fringes is maximal if the neutral axes of the plate are at 45° of parallel or crossed polarizers.

For crossed polarizers, the intensity at the output of the analyzer is:

$$I = I_0 \sin^2\left(\frac{\varphi}{2}\right) = I_0 \sin^2\left(\frac{\pi\delta}{\lambda}\right)$$

You can see the extinction of all wavelengths for which the optical path difference introduced by the plate is an integer multiple of the wavelength, ie $\delta = k\lambda_k = (n_e - n_o)e$.

For parallel polarizers,

$$I = I_0 \cos^2\left(\frac{\varphi}{2}\right) = I_0 \cos^2\left(\frac{\pi\delta}{\lambda}\right)$$

You can see the extinction of all wavelengths for which the optical path difference introduced by the plate is a half integer multiple of the wavelength, ie $\delta = (k + 1/2)\lambda_{k+1/2} = (n_e - n_o)e$.

The measurement of the wavelength of two successive dark fringes (corresponding to k and $k + 1$) or of two dark and bright successive fringes (corresponding to k and $k + 1/2$) allows in principle to determine simply the value of k (by solving an equation with one unknown). But beware, the variation of birefringence with wavelength, even if it is small, sometimes makes this determination difficult: we never find an integer value of k! Remember this and use the fact that the birefringence $n_e - n_o$ decreases with increasing wavelength (Cauchy's law) to determine k (by solving an inequality with one unknown).

Practical method to check each studied plate:

- Enter into an Excel spreadsheet, in ascending or descending order, all the measured values of the wavelengths corresponding to the dark fringes between crossed and parallel polarizers.

- Then determine the value of k, **positive integer,** for each dark fringe.

- Calculate the optical path difference for each dark fringe and plot the OPD as a function of wavelength: $\delta(\lambda) = k\lambda_k$ or $(k + 1/2)\lambda$.

- Check the consistency of your measurements and calculations, in particular the expected decrease of the OPD with the wavelength.

Note To check the value of k, positive integer, you can also use the following calibration points of the quartz birefringence:

$$\text{at } \lambda = 0.45\ \mu\text{m},\ n_e - n_o = 0.00937$$
$$\text{at } \lambda = 0.70\ \mu\text{m},\ n_e - n_o = 0.00898$$
$$\text{at } \lambda = 0.789\ \mu\text{m},\ n_o = 1.5442 \text{ and } n_e = 1.5533.$$

⇝ Carefully measure all the observed (and relevant) dark fringes **for polarizers first crossed and then parallel**. Determine the values of k corresponding to each dark fringe.

Q3 Explain why the value of k is easy to determine if there are very few dark fringes (less than 2).

Q4 For all plates studied, plot the optical path difference as a function of wavelength.

Q5 Deduce the value of the optical path difference at $546.1\ \text{nm}$ (green line of mercury).
Ask the teacher to check your results.

3 Babinet compensator

A Babinet compensator is made of two birefringent prisms glued together (see figure below). The extraordinary axis of the second prism is oriented along the ordinary axis of the first prism in order to compensate its birefringence. As a result, the overall birefringence introduced by the Babinet is directly proportional to the path length difference between the two prisms. Therefore, the birefringence varies linearly with the position of the Babinet along the x axis.

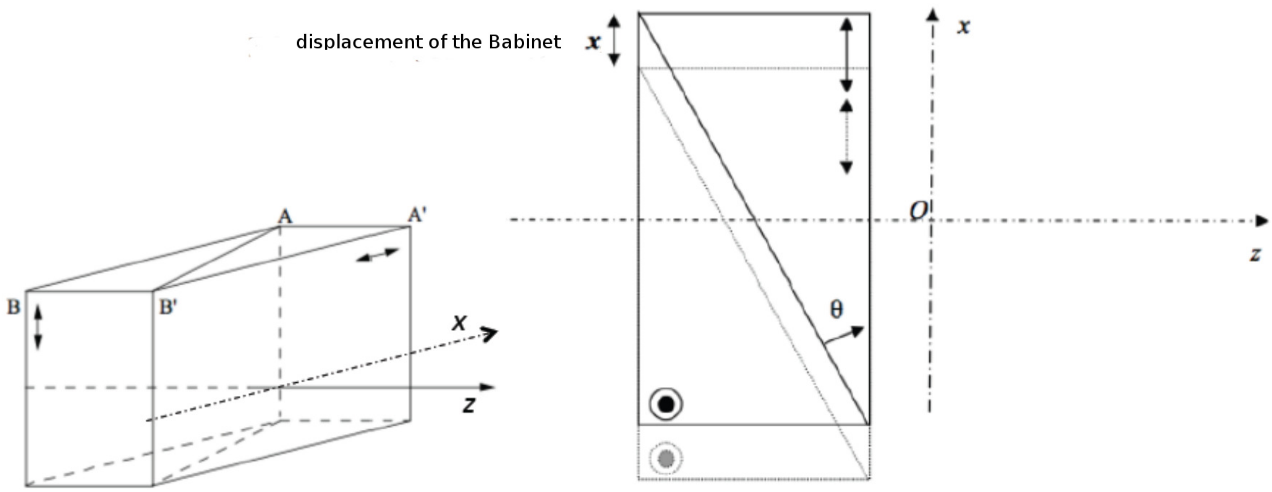


Figure 2.2 – Babinet compensator

For a displacement x, the optical path difference can be written:

$$\delta_\lambda(x) = 2[n_e(\lambda) - n_o(\lambda)]\tan(\theta)x = K_{cal}(\lambda)x$$

Note that $\delta(0) = 0$.

Let us consider a Babinet compensator between crossed polarizer and analyzer. Its neutral axes are at $45°$ with respect to the polarizer and the analyzer axis for maximum contrast. You can then observe interference fringes equidistant and parallel to the edge of the prisms (Oy) whose interfringe in the plane xOy is equal to:

$$i(\lambda) = \frac{\lambda}{2[n_e(\lambda) - n_o(\lambda)]\tan(\theta)} = \frac{\lambda}{K_{cal}(\lambda)}$$

Under white light illumination, there are fringes following Newton's color scale with white central fringe (for $\delta = 0$) between parallel polarizers or with black central fringe between crossed polarizers.

Method for measuring birefringence: If we add between the polarizer and the analyzer a birefringent sample whose neutral axes are parallel to those of the Babinet compensator, the fringes move proportionately to the additional OPD introduced by the sample. We can then measure the shift of the Babinet compensator required to bring the central fringe back in the center of the field and deduce directly the optical path difference introduced by the sample. The transverse displacement of the Babinet compensator is measured on the vernier of the micrometer screw with high precision.

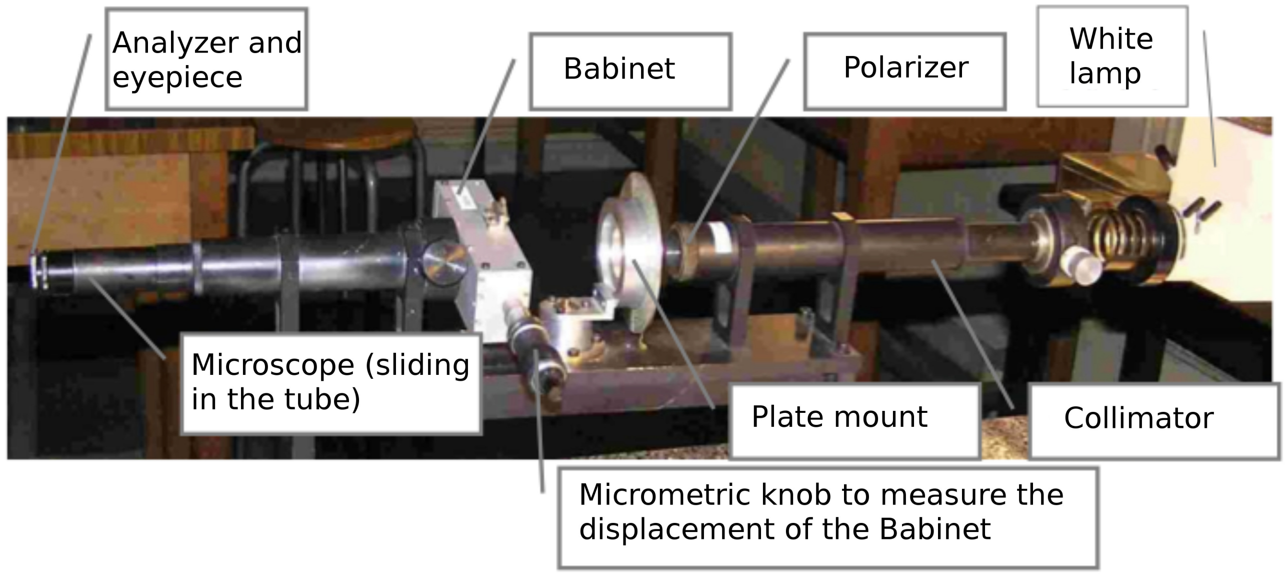


Figure 2.3 – Experimental setup

3.1 Settings

⇝ Illuminate properly the Babinet compensator. Direct illumination of the slit of the collimator by the lamp, without condenser, is sufficient to cover the entire aperture of the Babinet compensator, as long as you place the lamp close enough to the slit and you orient it properly.

⇝ Without compensator, cross polarizer (located at the end of collimator) and analyzer (attached to the microscope eyepiece). Install the compensator (it slides at the entrance of the tube containing the fixed front viewfinder).

⇝ Make the focus on the crosshairs etched (the viewfinder slides inside the tube). Turn the Babinet compensator to find the extinction and then turn it of $45°$. Fringes of high contrast should appear.

3.2 Calibration of the Babinet compensator

We can then **calibrate the Babinet compensator with monochromatic light** (mercury lamp equipped with a green filter). You need to measure as accurately as possible the interfringe (often called the Babinet compensator period).

Q6 Determine as precisely as possible the Babinet compensator interfringe at the wavelength of the green line of mercury. **This is about 2.4 mm. Repeat the measurement until you get close to this value.** Give the accuracy of your interfringe measurement.

This calibration allows the measurement of the optical path difference introduced by a plate at the wavelength of the green line of mercury (546.1 nm) (if this OPD is less than the maximum OPD measurable with the Babinet compensator).

Q7 What is the maximum measurable OPD with the Babinet compensator?

3.3 Measurement of the sample birefringence

⇝ Replace the mercury lamp with a white light source. The polarizer and analyzer are crossed and the direction of the axes of the compensator is at 45° with respect to the axis of the polarizers. Bring back the central dark fringe on the reticle. Press on the red button of the micrometer screw. This position will serve as a reference.

⇝ Place the plate in order to keep the dark fringe centered.

Q8 Explain why you align its neutral axes with the axis of the analyzer and polarizer.

⇝ Turn the plate by $45°$ around the optical axis. The black fringe is no longer centered. You must translate the compensator to bring back the dark fringe in the center. While doing so, check if the successive colors that you observe are consistent with corresponding Newton's color scale.

Q9 Measure the displacement of the Babinet compensator to bring back the dark fringe centered. Explain how this measurement allows direct calculation of the optical path difference introduced by the sample at the calibration wavelength (546.1 nm).

Note: to have a dark fringe well contrasted in the presence of the plate, it is important that the axes of the plate are well aligned with those of the Babinet compensator.

Q10 Calculate the OPD introduced by the plate at 546.1 nm. Evaluate the accuracy of this measurement.

Q11 Check that the value obtained is consistent with the values obtained by the method of channeled spectrum.

Present to the teacher the principle of the two measurement methods.

4 Conclusions on the set of measurements

Q12 For each plate, make a summary of the results obtained by the three methods. Explain why, for some samples, some methods are not appropriate.

Q13 For each plate, draw the optical path difference as a function of wavelength with the bars of uncertainty.

Q14 For each method, evaluate the accuracy of the results.

Q15 Determine the thickness of each plate, assuming that it is indeed a quartz plate cut parallel to the optical axis.
We can use the variation of $n_e - n_o$ of quartz as a function of wavelength:

$$n_e - n_o = 8.678.10^{-3} + \frac{145.025}{\lambda^2} \text{ with } \lambda \text{ in nm.}$$

Lab 3

Analysis of polarization states using a rotating analyzer

At the end of the session, you will be able to :

- produce a desired polarization state using wave blades and polarizers
- analyze a polarization state using a quarter-wave plate and polarizer
- describe the ellipsometry measurement method
- measure the polarization state at the output of a blade using an ellipsometer
- measure the polarization state of a beam after reflection from a surface
- explain the influence of the angle of incidence on the polarization of the reflected beam model the reflected beam by a TE/TM decomposition

Answering the 3 preparation questions in advance will enable you to be more effective during the session.

Contents

An ellipsometer is a widely used industrial device in particular for characterizing thin film deposition (thickness, indices). The purpose of this session is to study the working principle of a *rotating analyzer ellipsometer*. Through the first part of this session, you will become familiar with this device. **An oral presentation will end this part (section 3).** The ellipsometer is used in a second part to determine the nature of the vibration transmitted through a birefringent plate or reflected on a metal surface as a function of the incidence angle and the nature of the incident electromagnetic wave.

1 Preparation

P1 Recall the definitions of the transverse magnetic, TM (also called P-polarization) and transverse electric TE polarization (S-polarization).

The **intensity** coefficient of reflection R_{TE} and R_{TM} versus the incidence angle are plotted on the graph on figure 3.1:

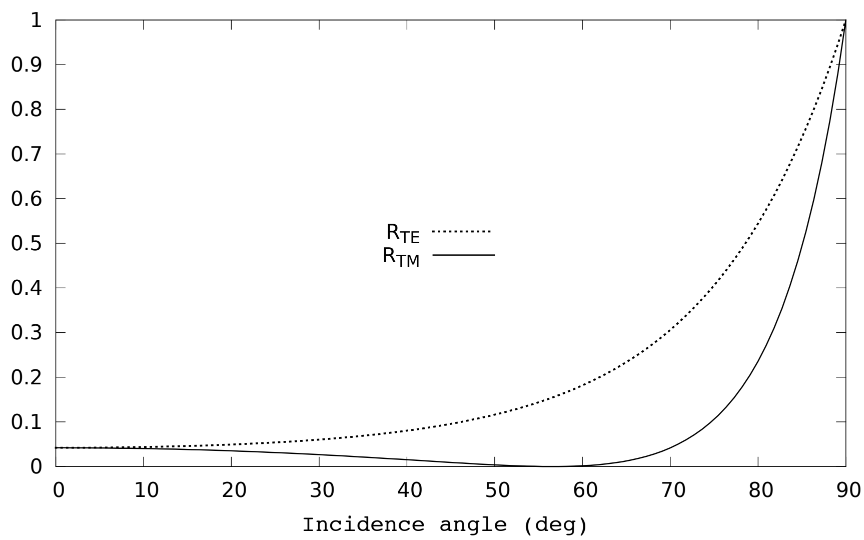


Figure 3.1 – Intensity coefficient of reflection R_{TE} and R_{TM} versus the incidence angle (standard glass $n = 1,515$ at $633\,\mathrm{nm}$)

P2 For what incident polarization is the intensity of the reflected beam minimal? Does this result depend on the angle of incidence?

P3 Calculate the value of Brewster's angle for a standard glass ($n = 1.515$ at 633 nm).

2 Experimental set-up

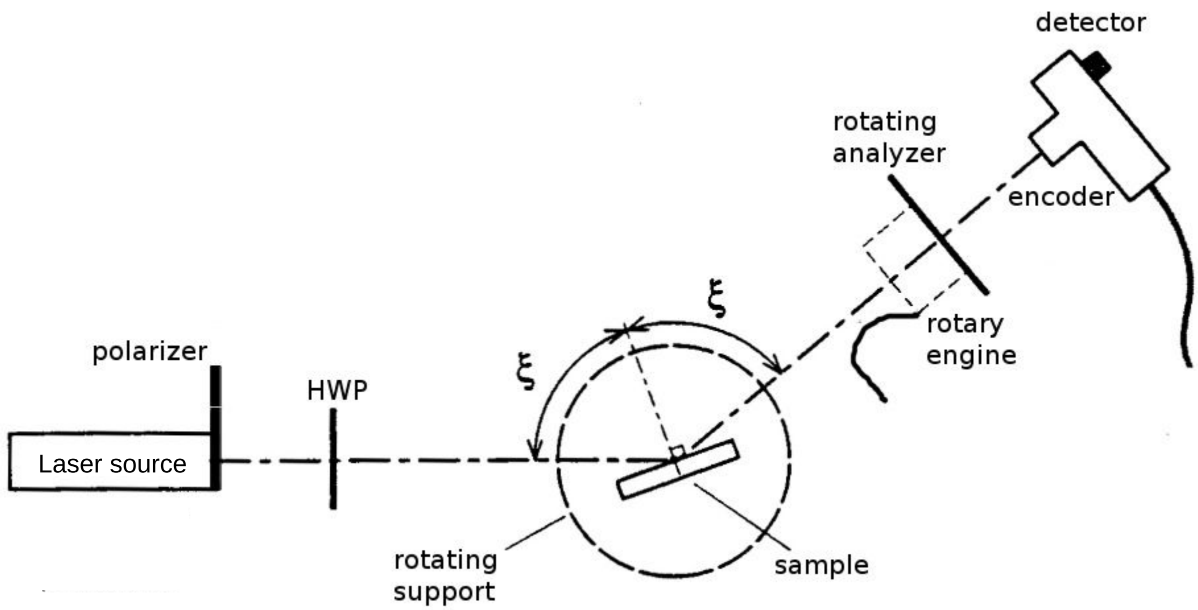


Figure 3.2 – Set-up scheme

The light source is a laser at $640\,\mathrm{nm}$, a polarizer is placed in the laser output.

The detector consists of a large sensitive area photodiode ($\varnothing \simeq 11$ mm) associated with a current-voltage amplifier carefully designed to have low noise and gain as constant as possible in its useful bandwidth. It has a selector for choosing between three sensitivities. The voltage delivered by the detector is assumed to be proportional to the received flux. First, we will visualize this signal with an oscilloscope.

The rotating analyzer consists of a linear polarizer, driven in rotation by an electric motor. Its rotational speed, displayed in revolutions per second on the LCD control unit of the encoder (the computer must be on), can be adjusted with the voltage of the drive motor.

An incremental encoder of angular position (orange box) is used to pinpoint the angular position of the analyzer. The model used delivers a signal, `TOP0`, which gives a single 'top' per turn, and a signal noted T, consisting of 4096 rising edges per turn. The `TOP0` signal frequency is denoted f_0 in what follows.

The incremental encoder plays an important role in the acquisition system of the signal you will use in the second part of the labwork.

3 Qualitative observation with the scope

We first use the red laser at $\lambda = 640$ nm, equipped with a polarizer at its output.

⇝ Carefully align the arm of the goniometer supporting the rotating analyzer and the detector on the direction of the laser beam. Turn on the power supply of the detector and the computer. Send the output signal of the detector on the oscilloscope. Also send the signal `TOP0` on the second channel of the oscilloscope and trigger the sweep of the oscilloscope on this signal (very narrow 5V TTL signal).

⇝ Apply power to the drive motor of the rotating analyzer with the DC supply provided for this purpose and set the speed of rotation (viewed on the display of ENCODER MANAGEMENT BOX) to about ten turns per second. **(Avoid exceeding 25 turns/s).**

⇝ Choose the sensitivity of the detector to have a signal sufficiently strong (several volts) but not saturated on the oscilloscope. The detector delivers a voltage proportional to the normal light flux it receives apart from a very low dark voltage. (The polarizer can, if necessary, be used to limit the flux hitting the detector to avoid a saturation).

Q1 Explain the observed signals on the oscilloscope with Malus' law, especially the relationship between the frequency of the signal from the detector and the frequency f_0 of `TOP0`.

⇝ Insert a half-wave plate (HWP) at 640 nm on the beam path and make the linear polarization rotate. Observe the changes of the signal from the detector viewed on the oscilloscope.

Q2 Interpret the signal obtained as a function of the orientation of the HWP. Take particular note of the direction of travel of the signal when you rotate the HWP and explain it.

⇝ Remove the half-wave plate and replace it by a quarter-wave plate.

⇝ Insert now a quarter-wave plate (QWP) on the linearly polarized beam.

Q3 Interpret the signal obtained as a function of the orientation of the QWP. How can the quality of this wave plate be checked. What is the influence of a non-normal incidence of the laser beam?

Q4 Fill in the table shown in the appendix below to illustrate the one-to-one correspondence between the contrast γ and phase Φ of the observed sine wave, and the polarization state (ellipticity ε, orientation θ of the ellipse) of the light which is incident on the rotating analyzer.

Q5 What is the mathematical relationship between γ and ε? And between Φ and θ?

4 Quantitative measurements with the computer

Simple measurements of phase and amplitude of a sine wave on the oscilloscope allow, strictly speaking, to determine the polarization state of a completely polarized beam hitting the rotating analyzer. However, these measurements are tedious and not very accurate. They can be very usefully computer-assisted. The objective of the next part is to get started with the acquisition software of ellipsometry measurements.

4.1 Using the acquisition and processing software

⇝ Remove the QWP before you move on.

⇝ Run the VI (Virtual Instrument) called
`Rotating Polarizer v2018.VI`.

This VI can acquire the detector signal, simultaneously view the detector signal and its FFT and calculate the parameters useful for polarimetry.

The sinusoidal signal detected can easily be written as:

$$S(r) = V_0[1 + \gamma \cos(4\pi r + \Phi)]$$

where r is the angular position expressed *in turns*, V_0 the average value, Φ the phase at origin and γ the modulation rate. The average value of the signal being arbitrary (it depends on the laser power and detector sensitivity), the information on the polarization is only contained in the modulation rate γ and the phase Φ. The Fourier transform of $S(r)$ whose argument is an *angular frequency* in turns^{-1} has a continuous component at *0 turn*$^{-1}$ of amplitude V_0 and a sinusoidal component at *around 2 turns*$^{-1}$ of amplitude $V_0\gamma/2$ and phase Φ:

$$S(r) = V_0 + \frac{\gamma V_0}{2} e^{i(4\pi r + \Phi)} + \frac{\gamma V_0}{2} e^{-i(4\pi r + \Phi)}$$

The two values γ and Φ are extracted in the LabVIEW program by fast Fourier transform (FFT) of the detector signal on *integer* numbers of revolutions of the analyzer. You will use the normal operation mode in the following (*orange button in the External clock position*). In that case, the acquisition of the signal $S(r)$ is synchronized with the encoder (rising edges of T). The amplitude $\gamma V_0/2$ and phase Φ values to be measured are properly calculated because the acquisition is perfectly synchronized with the sine signal at **2 turns**$^{-1}$.

Note about the Fourier Transform: In fact, as a result of non-uniformities of the rotating analyzer which introduce small distortions reoccurring at every turn, the actual signal is periodic in turn, except for some fluctuations and measurement noise, and it is decomposed into Fourier series having its fundamental at ± 1 turn $^{-1}$ and harmonics at ± 2, ± 3, ... turns^{-1}. Its FFT therefore presents peaks at these particular frequencies standing out from residual background noise. The peaks at 0 and ± 2 turn^{-1} are normally highly preponderant. The quantities γ and Φ, calculated as described above may be subject to random errors related to the noise signal and systematic errors due to deterministic imperfection of the analyzer.

⇝ Without any plate on the beam path, run the VI and observe the different results displayed: acquired signal, FFT, normalized amplitude and phase of harmonic 2.

Q6 Describe the spectrum, and identify the line(s) we are interested in. Discuss briefly the origin(s) of the other lines.

From this, the program computes the ellipticity ε and the orientation of the ellipse, and then plots it.

Q7 For a linear polarization, do we get a perfectly vanishing ellipticity? Why? Comment on the effect of stray light on the measured value of ε.

⇝ From now on, make sure to minimize stray light (using a colored filter and/or some black paper to shield the detector).

⇝ Using the HWP, check, by rotating the plane of polarization in a given direction, that the ellipse is plotted as the electric field of the light would be seen by an observer looking towards the light source.

4.2 Preparing a state of polarization

Here we will show that using a HWP and a QWP, starting from an initial linear polarization, one can prepare any polarization state (with arbitrary ellipticity and orientation).

Q8 First, how can one locate the neutral axes of the plate?

⇝ Then, using only a QWP, prepare a polarization state with a given ellipticity, say $\varepsilon = 30°$, oriented at $\theta = \varepsilon$.

⇝ Finally, use the HWP to change the orientation of θ arbitrarily, without changing ε.

4.3 Mesuring birefringence

⇝ Use this device to determine (modulo λ) the retardation induced by the unknown plate (a piece of adhesive tape on a microscope slide). To do so, first identify the neutral axes of the plate, and then orient them at $45°$ of the incident linear polarization.

5 Circular birefringence

⇝ The initial polarization is now linear. Put the quartz plate (sample labeled `OCP445`, thickness $L = 1.0$ mm) in the path of the beam.

Q9 What is the state of polarization at the output?

Q10 Does it change when the quartz plate is rotated around its axis? Does the plate behave like the waveplates you have studied before?

⇝ Now produce circular polarization before the plate.

Q11 What is the state of polarization after the plate?

One says that the plate shows *circular birefringence*. The angle of rotation of the polarization plane is $\alpha = \rho L$, where, to a good approximation $\rho \propto \lambda^{-2}$ (this is called Biot's law). For quartz, tabulated data give $\rho = \pm 22.09°/\text{mm}$ at the sodium D-line (589.3 nm). The $\pm$ sign arises from the fact that the rotation can occur to the left or to the right.

⇝ Come back to the situation of a linear incident polarization and measure the rotation angle α induced by the sample.

Q12 Check that the measured α and the data above are consistent with each other.

Q13 Is the quartz specimen is left- or right-rotatory?

⇝ Measure the rotation angle α induced by the thick quartz plate ($L = 7.7$ mm).

Q14 Is this specimen left or right-rotatory?

⇝ Use the green laser (with the polarizer!) to measure α at $\lambda = 532$ nm for both plates.

Q15 Is Biot's law fulfilled?

6 Reflection on a substrate

We use again the red laser.

So far, the *absolute* orientation of the polarization in the laboratory, e.g. with respect to the table, was not determined. To do so, we will use the phenomenon of polarization by reflection on glass at the Brewster angle ξ_B.

⇝ Using the glass plate (with a darkened back side to avoid reflections by the back interface) on the goniometer, find the appropriate incidence angle and the appropriate orientation of the incident linear polarization such that the reflected beam intensity vanishes.

Q16 Then, what is the orientation of the incident polarization with respect to the table? Record the corresponding value of the orientation of the ellipse given by the computer.

Q17 Give the experimental value of the Brewster angle that you measure, and use this to estimate the refractive index of the glass plate.

$\rightsquigarrow$ Show that for both S and P polarization, the output polarization remains linear whatever the angle of incidence ξ. (In practice, in order to have a strong enough signal, we restrict the range of measurments to $\xi > 60°$).

$\rightsquigarrow$ We now use a linear polarization at $45°$ (equal superposition of S and P).

Q18 Is the output polarization still linear? Does it depend on ξ?

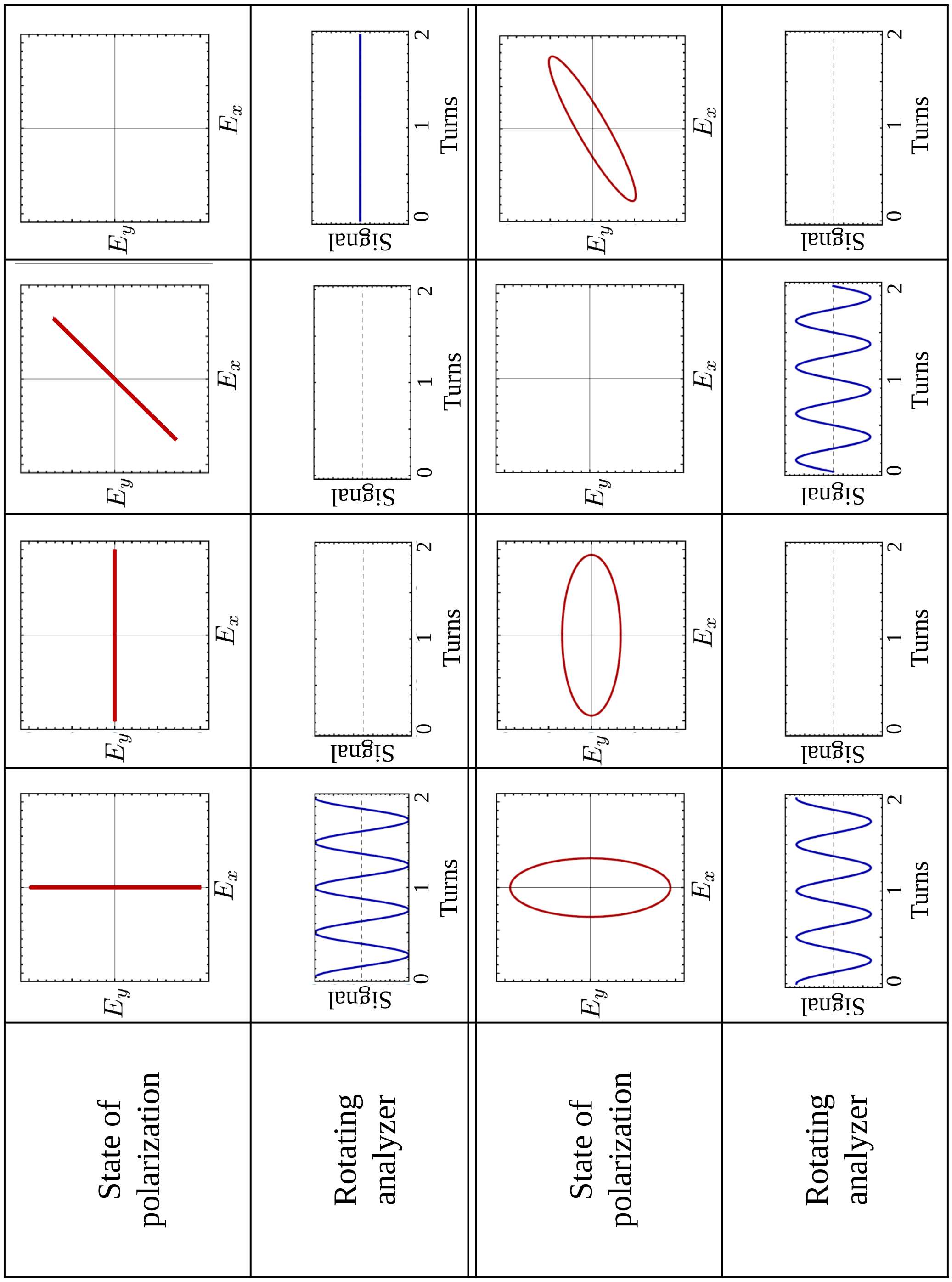
$\rightsquigarrow$ Perform the same experiments as in the previous question for the gold-coated mirror.

Q19 Is there a Brewster angle?

$\rightsquigarrow$ Do the same for the dielectric mirror (`BB1-E02` from Thorlabs).

Q20 For an incident linear polarization, in which conditions is the polarization of the reflected wave linear?

Appendix: correspondence between the state of polarization and the signal given by the rotating analyzer



Lab 4

Study of an electro-optic modulator

The aim of this session is to study the working principle of an electro-optic modulator and the way it is used. At the end of the session, you will be able to :

- align and set up an amplitude modulation bench using an OME
- interpret measurements/model with ellipses
- use and characterize the performance of a modulation bench
- Interpret interference in polarized light when using blades in non-collimated beams
- use the notions of ordinary and extraordinary polarization

Answering questions **P1** to **P4** in advance will help you to be more effective in the session.

Contents

1 Preparation: the electro-optic effect

The electro-optic effect will be seen during the second semester in lecture and in tutorials. The few items below allow us to carry out the labwork on and to understand the birefringence induced by an electric field and the use of an electro-optical component as an intensity modulator.

The application of an electric field on a non-centrosymmetric crystal can cause a change in the refractive index. If the change of index is proportional to the applied field, this phenomenon is called the Pockels effect (this is the case of the KD*P crystal studied in this lab). If, however, the change is proportional to the square of the applied field, this is called the Kerr effect.

The electro-optical effect is thus an effect of electrically induced birefringence. The crystal behaves as a birefringent plate with a slow axis and a fast axis whose indices vary depending on the applied voltage. We describe these variations by changing the index ellipsoid.

In an arbitrary coordinate system $Oxyz$ the equation of the index ellipsoid is:

$$\frac{x^2}{n_{xx}^2} + \frac{y^2}{n_{yy}^2} + \frac{z^2}{n_{zz}^2} + \frac{2xy}{n_{xy}^2} + \frac{2xz}{n_{xz}^2} + \frac{2yz}{n_{yz}^2} = 1$$

In the coordinate system $OXYZ$ of the medium neutral axis, one obtains:

$$\frac{X^2}{n_{XX}^2} + \frac{Y^2}{n_{YY}^2} + \frac{Z^2}{n_{ZZ}^2} = 1$$

The electro-optical effect results in a slight variation of the indices: the coefficients $1/n_{ij}^2$ undergo variations $\Delta(1/n_{ij}^2)$ and become the coefficients $1/n_{ij}'^2$:

$$\frac{1}{n_{ij}'^2} = \frac{1}{n_{ij\,(E=0)}^2} + \Delta\left|\frac{1}{n_{ij}^2}\right|$$

The variations of the coefficients $1/n_{ij}^2$ are calculated by taking the product of the (6 x 3) matrix of the electro-optical coefficients r_{ij}, which depend on the nature of the crystal, by the electric field vector, $\mathbf{E}$. For KD*P, which, without any applied electric field is a uniaxial crystal along Oz, the initial ellipsoid has the following equation:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1 \text{ with } n_o = 1.51 \text{ and } n_e = 1.47 \text{ at } \lambda = 0.6\mu\text{m}.$$

The symmetry properties of the KD*P crystal allow to show that the matrix of the electro-optic coefficients is:

$$\begin{pmatrix} \Delta\left[\frac{1}{n_{xx}^2}\right] \\ \Delta\left[\frac{1}{n_{yy}^2}\right] \\ \Delta\left[\frac{1}{n_{zz}^2}\right] \\ \Delta\left[\frac{1}{n_{yz}^2}\right] \\ \Delta\left[\frac{1}{n_{zx}^2}\right] \\ \Delta\left[\frac{1}{n_{xy}^2}\right] \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_Z \end{pmatrix}$$

with $r_{41} = 8.8.10^{-12}$m.V^{-1} and $r_{63} = 26.2.10^{-12}$m.V^{-1}

It can be easily shown that the ellipsoid of KD*P in the presence of an electric field E_z applied along Oz becomes:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2r_{63}E_z xy = 1,$$

where the appearance of a crossed term indicates a rotation of the ellipsoid. In the presence of an electric field along Oz, the neutral axes Ox' and Oy' are at 45° to the axis Ox and Oy.

Making a change of variables one can obtain the equation of the ellipsoid in these new neutral axes:

$$\begin{cases} x' &= \frac{1}{\sqrt{2}}(x+y) \\ y' &= \frac{1}{\sqrt{2}}(y-x) \end{cases} \Rightarrow \left(\frac{1}{n_o^2} + r_{63}E_z\right) x'^2 + \left(\frac{1}{n_o^2} - r_{63}E_z\right) y'^2 + \frac{z^2}{n_e^2} = 1$$

At first-order the equation of the ellipsoid in these new neutral axes can be written as

$$\frac{x'^2}{n_{x'}^2} + \frac{y'^2}{n_{y'}^2} + \frac{z^2}{n_e^2} = 1 \text{ with } \begin{cases} n_{x'} &= n_o - \frac{1}{2}n_o^3 r_{63}E_z \\ n_{y'} &= n_o + \frac{1}{2}n_o^3 r_{63}E_z \end{cases}$$

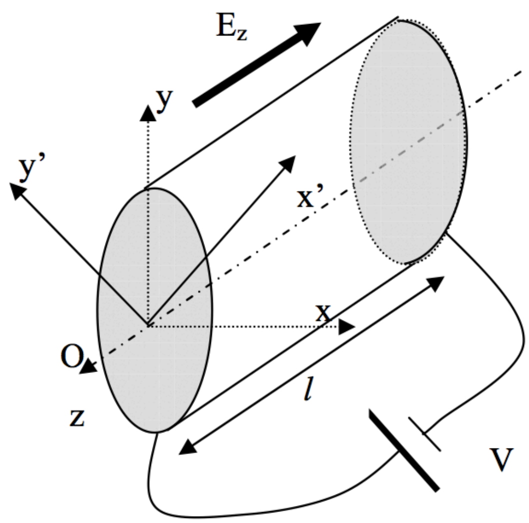


Figure 4.1 – Crystal sheme.

In the presence of a field E_z, we get thus two neutral axes Ox' and Oy' in the plane perpendicular to the z-axis, characterized by a birefringence:

$$\Delta n = n_{y'} - n_{x'} = n_o^3 r_{63} E_z$$

Let us consider a monochromatic plane wave linear along Oy propagating in the crystal along the direction Oz.

P1 Give the expression of the phase shift introduced and show that it is independent of the length l of the crystal. Give the expression of the voltage V_π for which the crystal behaves like a half-wave plate.

P2 Suggest a set-up using the KD*P crystal for modulating the amplitude of a linearly polarized electromagnetic wave.

P3 Suggest a set-up using the KD*P crystal for modulating the phase of a linearly polarized electromagnetic wave.

P4 Look for application examples of electro-optic modulators on the internet.

2 Caracterization of the electro-optic modulator

Align the following set-up:

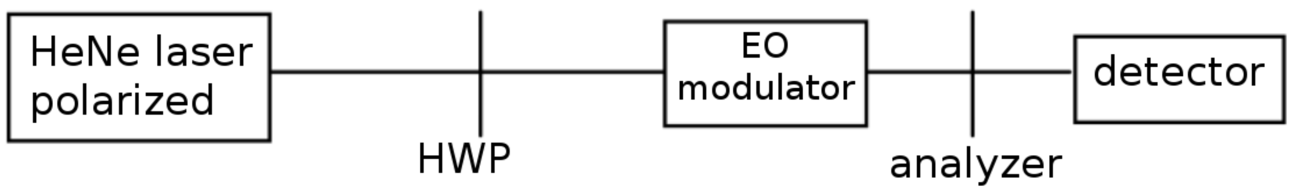


Figure 4.2 – Experimental setup

2.1 Setting the optical axis of the modulator with respect to the laser beam

⇝ Cross the analyzer with the linear polarization state produced by the half-wave plate (HWP).

⇝ Install and align the electro-optic modulator on the laser beam (for the moment, the modulator is not powered).

⇝ Observe the interference pattern obtained after the analyzer.

⇝ Adjust the orientation of the modulator in order to align the center of the black cross and the spot of the laser beam transmitted by the analyzer.

2.2 Setting when the modulator is powered

The high voltage power supply (HV) provides a voltage between 0 and 3000 V.

Warning High Voltage

- Never disconnect a cable when the power is on.
- Use only high-voltage coaxial cables (green) whose core is well protected.
- Never connect a high voltage cable directly to a low voltage cable.

The HV power supply can be adjusted with a potentiometer.

⇝ Apply to the crystal a DC high voltage close to 1500 V.

⇝ Orient the neutral axes of the crystal, Ox' and Oy', at 45° with respect to the analyzer axis. For this, two methods are possible:

1. Search the extinction by rotating the modulator around its axis. Then, starting from the extinction, turn the modulator by 45° around its axis.
2. Do not touch the modulator (in order not to misalign it). Search the extinction by rotating the half-wave plate and the analyzer. Then turn off the HV power supply and turn the analyzer by 45°, then rotate the half-wave plate to recover the extinction.

Q1 Explain and comment on the method of alignment you have chosen.

⇝ Visually check the intensity variation obtained by varying the voltage applied to the KD*P crystal.

2.3 Study of the characteristic of the transmitted flux as a function of the applied voltage

⇝ Measure the evolution of the light intensity exiting the analyzer with the HV (between 0 and 3000 V) applied to the modulator.

⇝ Evaluate the modulation rate obtained, defined by:

$$\eta = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

⇝ Measure the high voltage corresponding to the maximum transmission. For this voltage, precise the polarization state produced by the modulator. Explain why this voltage is called V_π.

Q2 Deduce for the measured value of V_π the value of r_{63}, taking into account the fact that the studied modulator is made of two identical crystals in series, subjected to the same field E_z, and whose phase differences add themselves.

2.4 Study of the polarization state produced by the crystal

Q3 For what value of the applied voltage is the electrooptic crystal equivalent to a half-wave plate? to a quarter-wave plate?

⇝ Check the polarization state at the exit of the crystal for these two voltage values, by explaining the method.

⇝ Apply a voltage of 700 V, and then of 1800 V to the crystal. For each case, determine the polarization state obtained. Find out the position of the major and minor axes of the ellipse and measure its ellipticity by a photometric measurement.

Q4 Deduce the phase difference introduced by the crystal and check that the measured phase difference is consistent with the expected values.

Present the obtained results to the teacher (oral presentation graded out of 5 points).

2.5 Use of the electro-optic crystal as a linear modulator of flux

Q5 Around what operating point of the characteristic previously obtained can the electro-optic crystal be used as a linear modulator of flux?

We will replace the power supply, that can not be modulated, by a low frequency generator. *In practice: a small blue box for low to high voltage adaptation is used to send the voltage delivered by the low frequency generator directly to the modulator.*
To be in the linear region of the characteristic, you can add a wave plate just before the modulator.

Q6 What kind of wave plate has the same effect as the previous high voltage applied to the modulator? How should one orient this plate to stay around the operating point chosen in the previous section?

Q7 Explain how to perform this setting.

⇝ Use this setup to send through the laser beam a modulation in the audio bandwidth from the tape player mini system.

Q8 Comment and interpret. Show your set-up to the teacher for validation.

⇝ Measure the modulation rate obtained for a voltage of 20 V peak to peak applied to the crystal. Check that this modulation rate is consistent with the characteristic obtained previously.

Q9 Comment on the evolution of the flux modulation when changing the orientation of the plate.

2.6 Black Cross?

⇝ Remove the waveplate. Observe the intensity distribution obtained at the output with the modulator switched off for the incident polarization and the crossed analyzer (black cross observed at the beginning of the session).

The black cross seen here is due to the existence of rays that do not propagate in the direction of the crystal's optical axis, i.e. converging or diverging rays, caused by parasitic scattering in the crystal. To analyze the propagation of a beam in a birefringent plate when we're not using parallel illumination, we suggest you make a few observations on an auxiliary bench.

3 Non-normal incident illumination on a birefringent sample.

Here we use a set-up that allows us to illuminate convergent white light on a spar crystal (negative uniaxial crystal) that is cut perpendicularly to the optical axis, and placed between crossed polarizers.

Q10 Describe the interference pattern that you observe.

⇝ Rotate the crystal around its optical axis. Then turn the polarizer and analyzer while keeping them crossed.

Reminder of propagation of light in uniaxal crystals. Polarisation is preserved only if the light polarisation direction is along one or the other (ordinary/extraordinary) direction.

- extraordinary wave electric field inscribed in the plane formed by the ray and the optical axis (here the axis of the optical bench),
- ordinary wave : electric field orthogonal to this same plane.

These two waves do not propagate at the same speed; they see a different index. The ordinary wave "sees" an index equal to the ordinary index n_o. The extraordinary wave "sees" an index (which depends on the angle θ of the ray) :

$$n" = \frac{1}{\sqrt{\frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}}}$$

This wave therefore propagates in a different direction (θ') :

$$\tan(\theta') = \tan(\theta)\frac{n_o^2}{n_e^2}$$

The diagram in figure 4.3 below shows the two rays in an arbitrary plane for a negative uniaxial crystal ($n_e < n_o \Rightarrow \theta' > \theta$) :

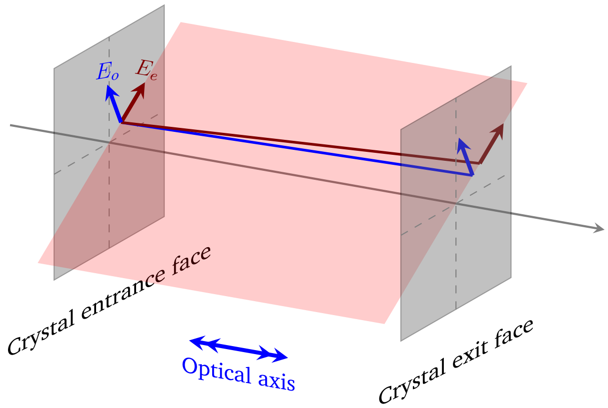


Figure 4.3 – Ordinary and extraordinary rays. The two rays are parallel at the crystal exit (not shown).

Q11 Deduce the origin of the black cross. Explain how the interference pattern is formed. In particular, explain why one finds Newton's color scale as one moves away from the white (or black) center of the pattern.

⇝ Place a QWP used to analyze a polarization state in the path of the light so that its neutral axis is at 45° from the black cross.

Q12 Explain why two black spots (corresponding to zero path-difference) appear on the interference pattern in the direction of the slow axis.

Appendices

Contents

1 Parameters describing an elliptic polarization

1.1 Definitions

Standard quantities defining elliptic polarisation appear on figure 4.4.

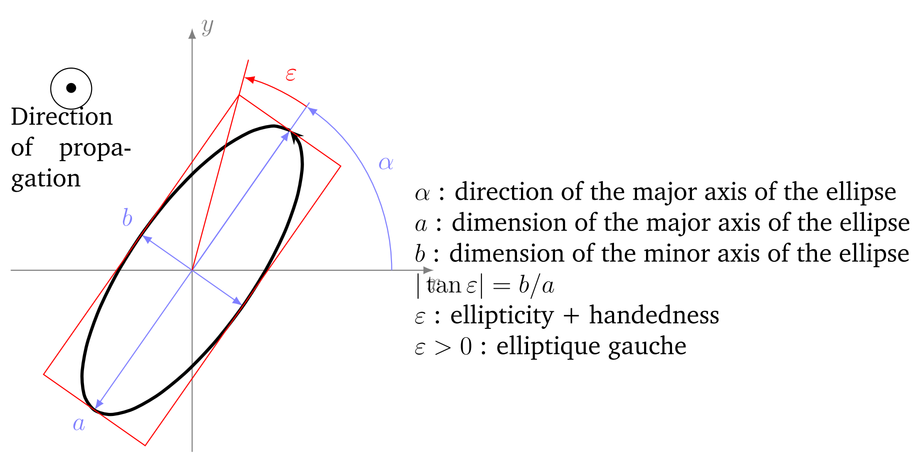


Figure 4.4 – Elliptic polarisation properties

1.2 Sense of rotation and dephasing

The projection of the electric field on the large (resp. small) axis denoted E_a (resp. E_b) are dephased by $\pm 90^\circ$. According to the sign of this phase, the ellipticity is left-handed or right-handed as it is illustrated in figure 4.5.

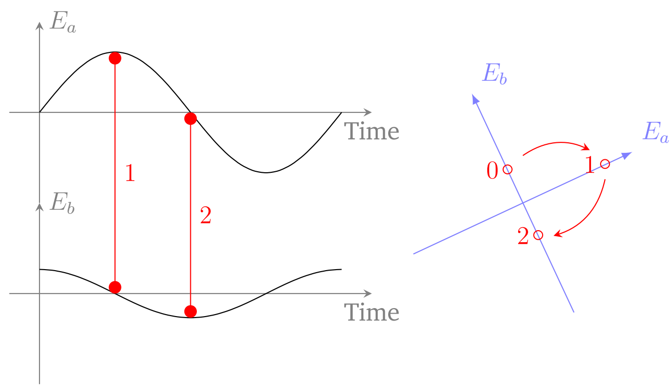


Figure 4.5 – Illustration of the relation between phase shift sign and sense of rotation of the ellipse.

1.3 Elliptic polarisation at the output of a quarter wave-plate

In particular, an elliptical polarization is created if a QWP is placed on the path of a linear polarized wave.

The ellipticity is therefore given by the angle between the direction of the input polarisation and the neutral axis of the QWP. Similarly to the diagram of figure 4.6, the ellipse is included in a rectangle whose diagonal lies on the input polarisation direction.

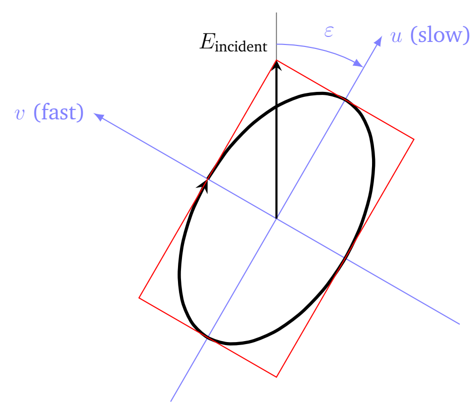


Figure 4.6 – Ellipse at the output of a quarter wave plate for a linear input polarisation. The ellipticity is indeed the angle between the direction of the input polarisation and the neutral axis of the QWP.

The axes of the ellipses are thus oriented in the direction of the neutral axes of the QWP.

1.4 Elliptical polarization at the output of an arbitrary birefringent plate oriented at 45°

In the case of a nondescript birefringent plate the polarisation is elliptical as well. However, nothing can be said in general about the direction of the ellipse axis. In the sole event where the incident input polarisation is oriented at 45°of the neutral axis, the ellipse axes are along the direction of the input polarisation.

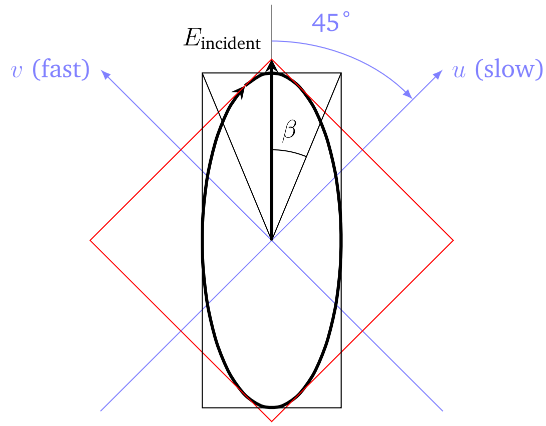


Figure 4.7 – Ellipse at the output of a nondescript birefringent plate.

The angle between a diagonal of the rectangle in which is included the ellipse defined by the extremity of the electric field vector and the direction of the incident polarisation is called β (see diagram on figure 4.7). The angle β fulfil the following relation :

$$\tan|\beta| = \left|\tan\left(\frac{\varphi}{2}\right)\right| \qquad \beta < \pi/2$$

where φ is the phase shift due to the plate (between projections on slow and fast axes).

On the previous scheme, β is rigorously equivalent to ε. But be careful ! In some cases, $\beta > 45°$. In that event, the large axis of the ellipse is orthogonal to the incident polarisation direction. The relation $\tan|\beta| = \left|\tan\left(\frac{\varphi}{2}\right)\right|$ is always valid but $\varepsilon = 90° - \beta$. In order to measure φ, it is sufficient to assess β. It is then possible to infer from β the output ellipticity properties.

Warning : Care must be taken when the polarisation is not at 45° from the plate axes. There, ellipticity and direction of the ellipse axes are not easily obtained as before !

On the following figure 4.8, phase shift introduced by the plate goes from 0 to 180° with a step of 15°. The resulting ellipticity is defined by the ellipse included in a square ($E_u = E_v$) and :

- ellipse axes are fixed at 90 °
- the ellipticity ε is given by $\pm\varphi/2\ [\pi/2]$)

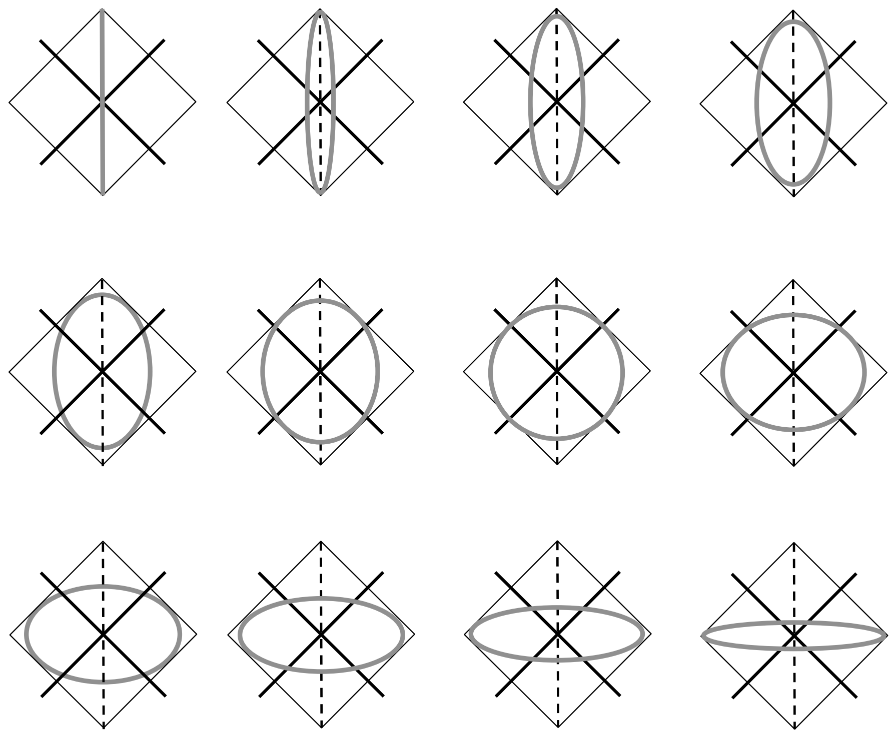


Figure 4.8 – Resulting ellipses with respect to the dephasing introduced by the birefringent plate. The incident polarisation is represented by a bold grey line when the phase is null and dashed otherwise. Its orientation stands at 45° from the neutral axes of the plate appearing in bold black lines.

2 Using a quarter waveplate to measure the ellipticity

It is a two steps method:

1. The small axis of the ellipse is spot thanks to a polariser.

2. A quarter-wave plate is added before the analyser with its slow axis perpendicular to the previously determined minor ellipse axis. This procedure allows to create a linear polarisation that makes an angle ε with the quarter wave plate slow axis. The extinction is recovered rotating the analyser by a ε angle. In this case the rotating angle is smaller or equal to 45°.

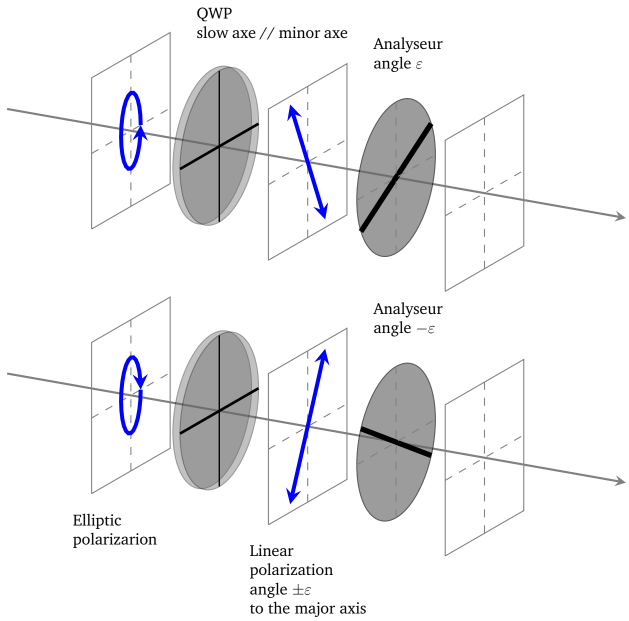


Figure 4.9 – Illustration for the quarter "waveplate method" to measure the ellipse's main parameters.

3 Newton's color scale

δ in nanometers optical path difference	scale with white center $I = I_0 \cos\left(\frac{\pi\delta}{\lambda}\right)$	scale with black center $I = I_0 \sin\left(\frac{\pi\delta}{\lambda}\right)$
0	white	black
40	white	iron-gray
97	yellowish-white	lavander-gray
158	yellowish-white	grayish-blue
218	brown yellow	clear gray
234	brown	greenish white
259	light red	almost pure white
267	carmin red	yellowish-white
275	dark brownish-red	pale straw-yellow
281	dark violet	straw-yellow
306	indigo	light yellow
332	blue	bright yellow
430	greyish-blue	brownish-yellow
505	bluish-green	reddish-orange
536	light green	red
551	yellowish-green	deep red
565	light green	purple
575	greenish-yellow	violet
589	golden yellow	indigo
664	orange	sky blue
728	brownish-orange	greenish-blue
747	light carmin red	green
826	purple	light green
843	violet purple	yellowish-green
866	violet	greenish-yellow
910	indigo	pure yellow
948	dark blue	orange
998	greenish-blue	bright reddish-orange
1101	green	dark violet red
1128	yellowish-green	light bluish-violet
1151	dirty yellow	indigo
1258	skin color	blue (greenish tint)
1334	brownish-red	sea green
1376	violet	bright green
1426	greyish violet blue	greenish-yellow
1495	greenish-blue	pink (light tint)
1534	blue green	carmin red
1621	pale green	carmin purple
1658	yellowish-green	violet grey
1682	greenish-yellow	greyish-blue
1711	greyish-yellow	sea green
1744	greyish-red mauve	bluish-green
1811	carmin	nice green
1927	reddish-grey	gris green
2007	greyish-blue	almost white grey
2048	green	light red
2338	light pink	light blue green
2668	light blue green	light pink