

# Lab work in photonics

## Quantum Photonics

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Master 2 QLMN  
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# P 1

## Entangled photons and Bell's inequality

*Please prepare P1 to P10 before the lab session.*

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## 1 Introduction

Quantum theory does not allow one to calculate the outcome of a measurement, but rather the probability of the different possible outcomes. Because of this probabilistic aspect, many physicist, including Einstein, were dubious and thought that quantum mechanics had to be an incomplete theory that was not accounting for the full reality. To illustrate this point, Einstein, Podolski and Rosen presented in 1935 a "gedankenexperiment" (mental experiment) where a measurement on an entangled two-particle state (Bell state or EPR state) lead to a paradox, which the authors interpret as a proof of the incompleteness of the quantum theory <sup>1</sup>.

For a Bell state, a measurement of the state of each particle taken individually gives a random outcome, but the results on the two particles are perfectly correlated. In other words, a measurement of the first particle state allows us to predict with certainty the state of the second one. For Einstein, Podolsky

<sup>1</sup>A. Einstein, B. Podolsky et N. Rosen, *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*, Physical Review **47**, 777 (1935)

and Rosen, the possibility of predicting the state of the second particle implies that this state exists before the measurement, implying that there is a set of “hidden” variables (in the sense that they are not described by the theory) that determine this state all along the experiment.

For a long time, it has seemed that the debate between an hidden variable interpretation and a purely probabilistic theory was a philosophical discussion. However, in 1964, John Bell <sup>2</sup> showed that there were some cases where the two interpretations were leading to incompatible observations, and that it was possible to settle the debate with an experiment. But it has been necessary to wait 15 more years for the progress in **quantum optics** to implement this experiment with pairs of entangled photons. The first unambiguous results have been obtained at the Institut d'Optique by Alain Aspect, Philippe Grangier and Jean Dalibard <sup>3</sup>.

This lab work, inspired by **M. W. Mitchell** and **D. Dehlinger**, aims to produce a two-photons Bell state, and allows you to determine, with your own measurements, which theory can be invalidated.

## 2 Bell states

### 2.1 Polarization state of a photon in quantum mechanic

When we measure the polarization of a photon with a vertical analyzer, we refer to the basis formed by the vertical polarization ( $|V\rangle$  parallel to the analyzer axis) and the horizontal polarization ( $|H\rangle$  orthogonal to the analyzer axis). In this basis, the polarization state is written:

$$|\psi\rangle = c_V|V\rangle + c_H|H\rangle. \quad (1.1)$$

The coefficients  $c_V$  and  $c_H$  are complex number such as  $|c_V|^2 + |c_H|^2 = 1$ . The measurement of the photon polarization can only gives two outcomes:

- The photon is transmitted by the polariser and its polarization state is projected on  $|V\rangle$ . Quantum mechanics predicts that the probability of this result is  $P_V = |\langle\psi|V\rangle|^2 = |c_V|^2$  ;
- The photon is blocked by the polariser and its polarization state is projected on  $|H\rangle$ . The associated probability is  $P_H = |\langle\psi|H\rangle|^2 = |c_H|^2$ .

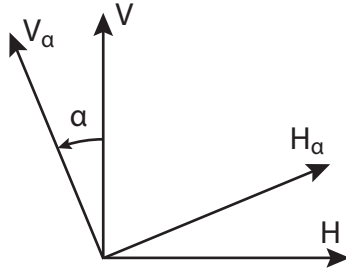
**P1** Write the polarization state of a photon with a rectilinear polarization, at an angle  $\alpha$  from the vertical. What is the probability to measure it in the state  $|V\rangle$ ?

<sup>2</sup>J. S. Bell, *On the Einstein-Podolsky-Rosen paradox*, Physics **1**, 195 (1964)

<sup>3</sup>See for instance : A. Aspect, J. Dalibard et G. Roger, *Experimental Test of Bell's Inequalities Using Time-Varying Analyzers*, Physical Review Letters **49**, 1804 (1982)

**P2** Write the polarization state of a photon with a left-circular polarization. What is the probability to measure it in the state  $|V\rangle$ ?

If we choose to measure the polarization state of a photon with a polariser rotated by an angle  $\alpha$  from the vertical, the new basis of the polarization state is  $\{|V_\alpha\rangle, |H_\alpha\rangle\}$  (figure 1.1).



**Figure 1.1:** The basis of the polarization state  $\{|V_\alpha\rangle, |H_\alpha\rangle\}$  is obtained by rotating the basis  $\{|V\rangle, |H\rangle\}$  by an angle  $\alpha$ .

**P3** Consider a photon in the polarization state  $|V\rangle$ . Write its state in the basis  $\{|V_\alpha\rangle, |H_\alpha\rangle\}$ . What is the probability  $P_{V_\alpha}$  of measuring the polarization  $V_\alpha$ ?

## 2.2 Pairs of polarization-entangled photons

During this lab work, you will produce and characterize pairs of polarization-entangled photons. The polarization state of those pairs is a Bell state, written as follow:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|V, V\rangle + |H, H\rangle) . \quad (1.2)$$

It is a non separable state, which means that we can not assign a polarization state to each photon individually.

**P4** Show that the probability  $P_V$  of measuring the photon 1 **or** the photon 2 in the polarization  $V$  is  $1/2$ .

**P5** What is the probability of measuring the two photons in the same polarization state ( $V, V$  or  $H, H$ )? What is the probability of measuring the two photons in orthogonal polarization states ( $V, H$  or  $H, V$ )?

In other words, a polarization measurement on one of the photons gives a random result, but if we measure the polarizations of the two photons of the pair, the results are always **correlated**. More generally, the probability to measure simultaneously the photon 1 in the polarization state  $V_\alpha$  and the photon 2 in the polarization state  $V_\beta$  is given by

$$P(V_\alpha, V_\beta) = |\langle V_\alpha, V_\beta | \psi \rangle|^2 . \quad (1.3)$$

**P6** Show that  $P(V_\alpha, V_\beta) = \cos^2(\alpha - \beta)/2$ . How does this probability change when the two axis of analysis are rotated by the same angle?

**P7** Show that the Bell state has the same form than (1.2) no matter what the basis of analysis is, which implies that:

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|V_\alpha, V_\alpha\rangle + |H_\alpha, H_\alpha\rangle) , \quad \forall \alpha . \quad (1.4)$$

This **rotational symmetry** of the polarization state is a crucial property of the Bell state that we will use to reveal the full extent of the correlation between the entangled photons.

## 2.3 Bell's inequality

For arbitrary orientated analyzers, the degree of correlation between the results of the measurements on the two photons can be quantified by the quantity

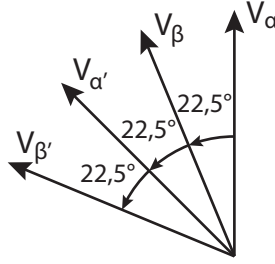
$$E(\alpha, \beta) = P(V_\alpha, V_\beta) + P(H_\alpha, H_\beta) - P(V_\alpha, H_\beta) - P(H_\alpha, V_\beta) . \quad (1.5)$$

From this quantity, we can define the **Bell parameter** :

$$S(\alpha, \alpha', \beta, \beta') = E(\alpha, \beta) - E(\alpha, \beta') + E(\alpha', \beta) + E(\alpha', \beta') . \quad (1.6)$$

It is that parameter that allows one to distinguish between the hidden variable theory and the quantum theory. Indeed, Bell showed that according to the hidden variable theory,  $S$  should be lower than 2 no matter the state of the two photons. It is the so-called **Bell inequality**. On the other hand, the quantum theory predicts a value of  $S$  strictly greater than 2 for a Bell state, for a certain choice of angles of analysis.

**P8** Show that  $S = 2\sqrt{2}$  for the set of angles of analysis depicted in figure 1.2. In this configuration, the Bell inequality are maximally violated by quantum mechanics.

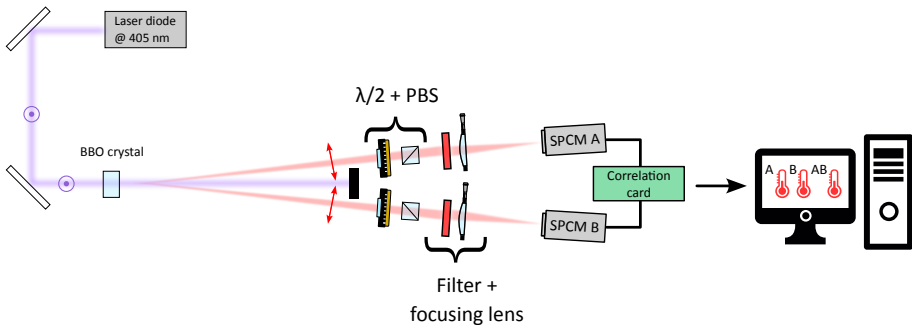


**Figure 1.2:** This set of angles of analysis maximizes the Bell parameter predicted by quantum mechanics.

### 3 Experiment : Measurement of Bell's parameter

#### 3.1 Global overview of the setup

The source of polarization-entangled photons consists in a laser diode emitting a 405 nm beam, vertically polarized and focused on two non-linear crystals of BBO. In those crystals, a parametric conversion process turn the 405 nm photons into a pair of 810 nm photons. One of the crystal generates pairs of photons with a vertical polarization, and the other one generates a horizontally-polarized pair. The twin photons are emitted symmetrically with respect to the probe beam, inside a cone with an aperture of about  $3^\circ$ . An half-wave plate working at 405 nm and a Babinet compensator allows one to adjust the polarization state of the probe beam.



**Figure 1.3:** Schematic diagram. A BBO crystal pumped by a laser diode creates two beams of correlated, or even polarization-intersecting photons. On each channel, the orientation of the analyzed polarization is adjusted using a half-wave plate. Photons are focused and detected by avalanche photodiodes, whose count and coincidence signals are displayed by the computer.

To collect the infrared photons, we use an avalanche photodiode (APD) on both side of the emission cone. Before entering into the detector, the infrared photons go through a polarization analyzer made of an half-wave plate, a polarization beam splitter (PBS), a lens to focus the beam onto the photodiode, and a interferential filter centered on 810 nm with a 10 nm width. A FPGA card is used to count the number of photons detected, as well as the number of coincidences, which correspond to a simultaneous detection on both arms. Those measurement are then displayed by a Labview code.

### 3.2 Single photon counting modules

The photodetectors are made of silicon avalanche photodiodes used in single photon detection mode. On each arm *A* and *B*, the detection of a photon produce a TTL pulse (0 V to 5 V) with a temporal width of 25 ns.

**Warning !** Those detectors are very, very expensive and would be destroyed by a strong photon flux! Always check that the black tubes and the filters are installed to protect the photodiodes. Wait for the teacher authorization to switch on the detectors, and make sure that the main lights are off and the door is closed.

### 3.3 Events and coincidences counters

The pulses and coincidences are counted by a Nucleo card collecting the pulses from the detectors, as well as the coincidence pulses generated by the output of a cascade of flip flop circuits. A coincidence pulse is generated when two pulses from the two distinct detectors reach the flip flop circuits within a coincidence window. The duration of this window is set by the time constant of RC circuits that set the reinitialization time of the flip flops.

**Q1** Why do the cables need to have the same length?

~ Make the following settings:

- Before doing anything else, switch on the power supply of the Nucleo card, start the Python program and initiate the connection to the Nucelo card.
- Switch off every lights and turn the photons counters on.
- Measure the number of dark counts. The lower this number is, the better the detectors are.
- Switch a weak light and check that the number of detected photons stays well below  $10^6$  photons/s.

### 3.4 Rate of accidental coincidences

For now, since the light sources are chaotic, the photons arriving in A and B are not correlated, and therefore the potential coincidences that you might detect are accidental. We note  $n_A$  and  $n_B$  the **counting rate** (average number of photons per second) on channels A and B,  $n_f$  the rate of accidental coincidences, and  $\tau$ , the duration of the coincidence window.

**Q2** Show that the rate of accidental coincidence is given by :  $n_f = n_A n_B \tau$ .

~ Measure the rate of accidental coincidences for the coincidence circuit.

**Q3** Calculate the duration of the coincidence window using the formula established in question **Q2**. Use this value to set the slider and the reference of the "corrected" coincidence gauge in the software.

This measure allows one to check the behavior of the coincidence counters. Call the teacher if the results are way different from the value of table ??.

### 3.5 Pump diode

The pump diode is a 405 nm laser with about 60 mW of output power. The light emitted by the diode is linearly polarized. The wearing of security glasses is mandatory!

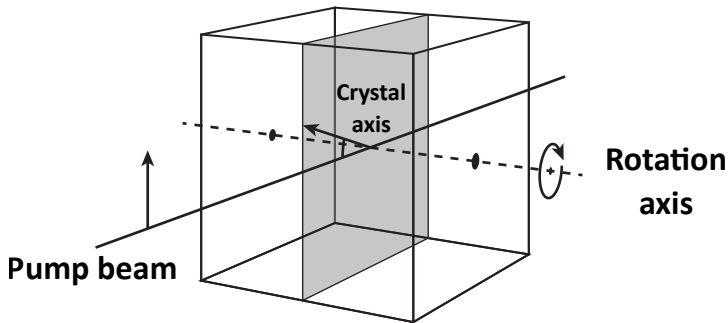
↪ Make the following settings:

- Press the two buttons to turn on the temperature regulator of the diode. The temperature is already set to obtain the correct wavelength. Don't try to change it.
- Press the two buttons to turn on the current supply of the diode, and set the current at maximum (about 95 mA).
- If they are present on the bench, remove the optical elements mounted before the BBO crystals (half-wave plates at 405 nm and Babinet compensator) as well as the half-wave plates at 810 nm and the PBS cubes in front of the detection channels.
- Check the alignment of the beams (they should be already well aligned).

### 3.6 Parametric conversion

The photon pairs are produced by parametric down conversion in non-linear crystals  $\beta$ -BaB<sub>2</sub>O<sub>4</sub> (baryum  $\beta$ -borate, BBO for short). During the non-linear process, a photon of the 405 nm pump can be converted in a pair of twin 810 nm photons. The BBO is a negative uniaxial birefringent crystal. We use a type I phase matching, meaning that the twin photons have the same polarization.

The pump beam is orthogonal to the entrance plane of the crystals. The optical axis of the crystals makes an angle of about 29° with the pump beam. The optical axis of the first crystal and the axis of the pump are forming a vertical plane, and the 810 nm twin photons emitted from this crystal are horizontally polarized (see figure 2.3). The optical axis of the second crystal and the axis of the pump are forming a horizontal plane, and the twin photons are emitted with a vertical polarization. In both cases, the twin photons are emitted symmetrically with respect to the pump axis, within a cone of about 3° of aperture.



**Figure 1.4:** The optical axis of the crystal is oriented “vertically” with respect to the pump axis. The other crystal is oriented “horizontally”.

**Note:** The direction of emission of the twin photons varies very quickly with the angle between the crystal axis and the pump axis. To optimize the number of twin photons arriving onto the detectors, one should carefully set the orientation of the two crystals, using the horizontal and vertical screws of the mount (see figure 2.3).

### 3.7 Collimation lens

To efficiently detect the pairs, the waist of the pump beam (which is inside the BBO crystals) is imaged onto the APD's sensitive surface. It is a difficult setting because this surface has a diameter of  $180\mu\text{m}$  only. The lenses have a focal length of 75 mm and a diameter of 12,7 mm. They are mounted 1040 mm away from the crystal.

**Q4** Calculate the position of the image of the waist.

↪ Check that the APD are approximatively at this position.

**Q5** What is the magnification? Give an estimation of the diameter of the waist, then check that the size of the sensitive area of the photodiode is not limiting.

### 3.8 Optimization of the coincidence number

The direction of emission of the twin photons is very sensitive to the orientation of the crystals, which needs to be carefully optimized. The pump beam being vertically polarized, only the crystal whose axis is in a vertical plane can satisfy

the phase matching condition for now. Its orientation can be adjusted thanks to the horizontal screw.

↪ Optimize the number of coincidences by tuning the screw.

**Q6** Calculate the number of accidental coincidences. Do we need to take it into account?

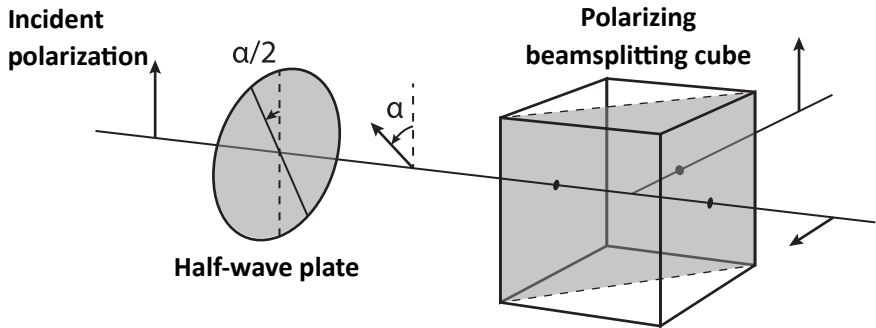
**Q7** Calculate the ratio between the number of coincidences and the number of photons detected on each channel. Comment.

↪ We will now optimize the orientation of the crystal whose axis is in an horizontal plane :

- Put the 405 nm half-wave plate before the BBO crystals.
- **Carefully identify the own axis of the plate (they do not correspond exactly to the 0 and 90° position). A good identification is crucial for the following.**
- Check that the number of coincidences is unchanged if the axis of the plate are horizontal and vertical. Explain why.
- Turn the plate at 45° to align the polarization of the pump beam with the horizontal axis.
- Optimize the number of coincidences by touching the vertical screw of the crystals.

### 3.9 Polarization analyzer

↪ Put the 810 nm half-wave plates and the PBS cubes on each channel. Together they form a polarization analyzer (see figure 1.5).



**Figure 1.5:** The polarization analyzer is made of an half-wave plate (whose orientation is adjustable) and a PBS cube.

**Warning :** The angle of the analyser is equal to the angle of analysis divided by two. So be careful to distinguish the angle of analysis from the angle of the plate!

**Q8** Show that when the plate is vertical, the analyzer transmits the horizontal polarization, and therefore allows to detect the  $|H\rangle$  photons. How much do you need to turn the plate to detect  $|V\rangle$  photons?

~ Settings and adjustments:

- Check experimentally that the twin photons are in the state  $|H\rangle_1 |H\rangle_2$  if the 405 nm half wave plate is vertical, and in the state  $|V\rangle_1 |V\rangle_2$  if the plate is rotated by  $45^\circ$ .
- If necessary, turn again the screws of the BBO crystals to have approximately the same number of coincidences for those two positions of the plate.
- Then set the plate to  $22,5^\circ$  (try to be very precise) to obtain approximately the same number of coincidences with the two analyzers in vertical position and the two analyzers in horizontal position (it is impossible to obtain a perfect equality, but try to balance the coincidences as much as you can).

~ Measure the number of coincidences when the two analyzers are parallel (horizontally or vertically). Measure the number of coincidences when the two analyzers are orthogonal.

↪ Adjust the angle of the half-wave plate to obtain the same rate of coincidence in both situations.

↪ Measure the coincidence rates when the two analyzers are parallel and perpendicular in the diagonal base (at  $45^\circ$ ).

↪ Note the absolutely astonishing result of this measurement!

**Q9** What results were you expecting? Compare with the previous measurement.

The previous settings produced a source of photon pairs created in either crystal equiprobably. This is a pair of entangled photons! Now you need to precisely adjust the polarisation state of the pump beam to obtain a Bell state.

### 3.10 Realization of a Bell state

For now, your twin photons are in the state

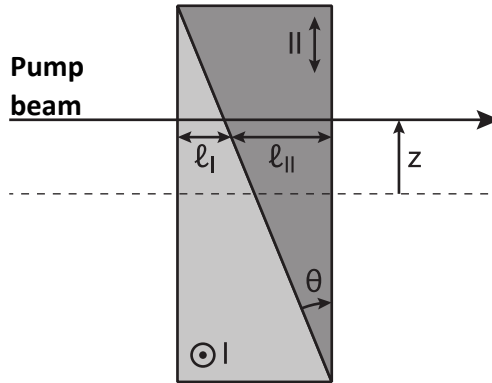
$$|\psi\rangle = \frac{1}{\sqrt{2}} (|V, V\rangle + e^{i\phi}|H, H\rangle) . \quad (1.7)$$

Where  $\phi$  is the phase difference between the twin photons horizontally polarized, and the twin photons vertically polarized. This phase difference is linked to the birefringence of the BBO crystals.

**Q10** Calculate the join probability  $P(V, V)$  and  $P(H, H)$  for the state (1.7). Does it allows you to distinguish the state (1.7) from the Bell state (1.2)?

**Q11** Show that  $P(V_{45^\circ}, V_{45^\circ}) = (1 + \cos(\phi))/4$ . What is the difference with the Bell state?

To obtain a Bell state instead of the state (1.7), you need to compensate the dephasing  $\phi$ , by using a Babinet compensator. If the axis of the compensator match the horizontal and the vertical direction, you can translate it orthogonally to the pump axis to linearly vary the dephasing between the two polarisation components (see figure 1.6). For the pump wavelength, this dephasing typically varies by  $2\pi$  for a translation of about 5 mm.



**Figure 1.6:** The Babinet compensator consist in two pieces of uniaxial birefringent crystal. The pieces are beveled and mounted one against the other, in such a way that the optical axis of the two crystals are orthogonal (and both parallel to the entrance plane). If we note  $n_1$  the refractive index along the optical axis, and  $n_2$  the refractive index along the direction orthogonal to the optical axis, we see that the dephasing  $\phi_B = \phi_H - \phi_V$  accumulated between the polarisation components of a beam with a wavelength  $\lambda$  is given by  $\lambda\phi_B/2\pi = (n_2\ell_I + n_1\ell_{II}) - (n_1\ell_I + n_2\ell_{II}) = 2(n_2 - n_1) \tan(\theta) z$ .

~ Mount the compensator on the bench.

~ Check that the coincidence rate are still equal when the analyzers are parallel horizontally and vertically. If it is not the case, it means that the axis of the compensator do not match the vertical and horizontal directions, and you need to slightly rotate the compensator.

~ Put the analyzers at  $45^\circ$  on both channels and plot the coincidence rate as a function of the compensator translation over 10 mm (by step of 0,5 mm). Take an integration time of 10 s.

**Q12** Does the curve behave as expected? Comment on the contrast.

**Q13** What are the points of the curve corresponding to a compensation of the dephasing induced by the crystals?

### 3.11 Variation of the join probability

The two theories we want to test (quantum theory and local hidden variable theory) do not predict the same variation of the join probability  $P(V_\alpha, V_\beta)$  as a function of the relative angle  $\alpha - \beta$ . So it is interesting to measure it. Experimentally, we can estimate the probability  $P(V_\alpha, V_\beta)$  by setting the analyzers at  $\alpha$  and  $\beta$  and calculating the ratio between the coincidence rate and the photon rate on each channel.

**Important note:** Only complete the measurement below if you have enough time left before the end of the session. Otherwise, go directly to the Bell parameter measurement.

↪ Fix the angle of one of the two analyzers at  $0^\circ$  and plot the variation of the coincidence rate as a function of the angle of the second analyzer. During this measurement, check that the photon rate on each channel remains approximatively constant. Re-do the same measurement, but this time set the first angle to  $45^\circ$ .

**Q14** Compare those measurements to the prediction of quantum mechanics (question P6).

### 3.12 Measurement of Bell's parameter

You will now experimentally evaluate the Bell parameter defined by the relation (1.6), and whose value allows you to invalidate the local hidden variable theory. To do so, you will measure the join probabilities  $P(V_\alpha, V_\beta)$ ,  $P(H_\alpha, H_\beta)$ ,  $P(V_\alpha, H_\beta)$  and  $P(H_\alpha, V_\beta)$  for the following set of angles of analysis:  $\{\alpha, \beta\}$ ,  $\{\alpha, \beta'\}$ ,  $\{\alpha', \beta\}$  and  $\{\alpha', \beta'\}$  define on figure 1.2. To minimize the uncertainty of each measurement, one need to maximize the number of coincidences detected, so one needs to count them over a longer duration. Indeed, the fluctuations in the number of coincidences  $N_c$  detected over an interval  $T$  are linked to the photonic "shot noise", which has a Poisson statistics. This implies that the statistical uncertainty (standard deviation)  $\sigma[n_c]$  on the coincidence rate  $n_c = \langle N_c \rangle / T$  verifies :

$$\sigma[n_c] = \frac{\sigma(N_c)}{T} = \frac{\sqrt{\langle N_c \rangle}}{T} = \sqrt{\frac{n_c}{T}}, \quad (1.8)$$

so that the relative uncertainty

$$\frac{\sigma[n_c]}{n_c} = \frac{1}{\sqrt{\langle N_c \rangle}} = \frac{1}{\sqrt{n_c T}}. \quad (1.9)$$

**Q15** If you count an average of 100 coincidences/s, what is the standard deviation of the coincidence rate? What is the duration required to divide this standard deviation by 10?

↪ Fill the Excel chart on the computer for each of the 16 measurements. Take an integration time of 10 or 20 s to have a good precision. The Bell parameter is then automatically calculated.

**Q16** What value of the Bell parameter do you obtain and what are the error bars? Does your measurement allow you to invalidate the local hidden variable theory or the quantum theory?

**Q17** What result would allow you to invalidate quantum mechanics?



## P 2

# Two-photon interference : Hong, Ou and Mandel experiment

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## 1 Introduction

The Hong, Ou and Mandel (HOM ) experiment, conducted in 1987<sup>1</sup>, is the first observation of a quantum interference between two photons with no classical explanation (*i.e.* no explanation with a wave description of light). This phenomenon appears when two indistinguishable photons arrive simultaneously at the two input ports of a 50/50 beam splitter. The distribution of the two photons between the two output ports of the beam splitter exhibit a surprising behavior, that can not be explained with a classical theory of light...

### 1.1 The HOM effect

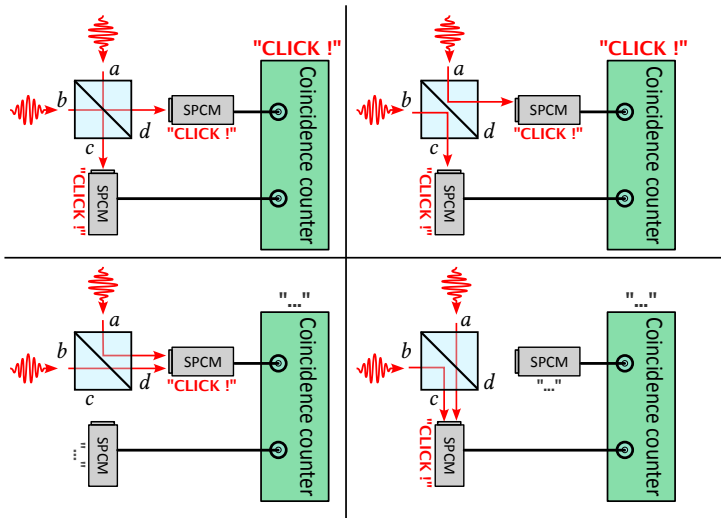
If a photon arrives on a 50/50 beam splitter, it has 1/2 probability of being transmitted or reflected. When two photons arrive simultaneously on a beam

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<sup>1</sup>C. K. Hong, Z. Y. Ou et L. Mandel, *Measurement of subpicosecond time intervals between two photons by interference*, Physical Review Letters **59**, 2044 (1987)

splitter, each one at a different input ( $a$  or  $b$ ), there are four possibilities (see figure 3.1) :

- The photon entered in  $a$  goes out in  $c$ , and the photon entered in  $b$  goes out in  $d$ ;
- The photon entered in  $a$  goes out in  $d$ , and the photon entered in  $b$  goes out in  $c$ ;
- Both photons go out in  $c$  ;
- Both photons go out in  $d$ .



**Figure 2.1:** Two indistinguishable photons arrive simultaneously on a beam splitter. Two single-photon detectors at the output  $c$  and  $d$  record which way the photons went out.

If the photons were classical particles, the 4 cases would always be equiprobable, in the case where the reflectivity and transmittivity of the beam splitter are equal. But according to the quantum theory, if the two particles are **indistinguishable** there is no way to know which particle is which at the output of the beam splitter, and therefore there is only 3 possible **observations** :

- we observe one photon in each output ;
- we observe two photons in output  $c$  ;

- we observe two photons in output  $d$ .

Then, according to quantum mechanics, the probability of observing a photon in each output results from the **interference** between the two **classical trajectories**  $(a, b) \rightarrow (c, d)$  and  $(a, b) \rightarrow (d, c)$ . It then depends on the **relative phase** associated to those trajectories.

In our case, two photons are **indistinguishable** when their properties (polarization, frequency, transverse mode) can not be distinguished with our experimental setup. Note that the two photons do not need to be identical, which would imply that they have identical properties, for them to be indistinguishable. For instance, two photons with different frequencies (or energies) behave as indistinguishable photons as long as the experimental setup does not allow one to measure the difference in their frequencies. As a consequence, establishing whether two photons are indistinguishable or not depends on the experimental setup which is used.

## 1.2 Formalism

In classical electromagnetism, a beam splitter is modelled by a **real unitary matrix**, that links the **electrical fields**  $\mathcal{E}_{a,b}$  of the inputs to the electrical fields  $\mathcal{E}_{c,d}$  of the outputs:

$$\begin{pmatrix} \mathcal{E}_c \\ \mathcal{E}_d \end{pmatrix} = U \begin{pmatrix} \mathcal{E}_a \\ \mathcal{E}_b \end{pmatrix} \quad \text{with} \quad U = \begin{pmatrix} t & r \\ -r & t \end{pmatrix}. \quad (2.1)$$

The unitarity property  $U^\dagger U = \mathbb{1}$  stands for the **energy conservation** between the input and the output of the beamsplitter. It leads to a relationship between the reflectivity and transmittivity coefficient of the beam splitter.

$$r^2 + t^2 = 1. \quad (2.2)$$

A 50/50 beam splitter corresponds to the case where  $r = t = 1/\sqrt{2}$ .

In quantum optics, the complex electromagnetic field is replaced by **creation and annihilation operators** :

$$\mathcal{E}_a \rightarrow \{\hat{a}, \hat{a}^\dagger\}, \mathcal{E}_b \rightarrow \{\hat{b}, \hat{b}^\dagger\}, \mathcal{E}_c \rightarrow \{\hat{c}, \hat{c}^\dagger\}, \mathcal{E}_d \rightarrow \{\hat{d}, \hat{d}^\dagger\}. \quad (2.3)$$

The beam splitter links those operators in the same way than with the electric fields :

$$\begin{pmatrix} \hat{c} \\ \hat{d} \end{pmatrix} = U \begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} \hat{c}^\dagger \\ \hat{d}^\dagger \end{pmatrix} = U \begin{pmatrix} \hat{a}^\dagger \\ \hat{b}^\dagger \end{pmatrix}. \quad (2.4)$$

The unitarity property of the matrix  $U$  ensures that **the number of photons is conserved** between the input and the output :

$$\hat{c}^\dagger \hat{c} + \hat{d}^\dagger \hat{d} = \hat{a}^\dagger \hat{a} + \hat{b}^\dagger \hat{b}. \quad (2.5)$$

The quantum state that corresponds to a situation where two photons enter in  $a$  and  $b$  is obtained by using the operators  $\hat{a}^\dagger$  and  $\hat{b}^\dagger$  on the state which corresponds to the **electromagnetic vacuum** :  $\hat{a}^\dagger \hat{b}^\dagger |\text{vacuum}\rangle$ . By inverting the relation 2.4, one can link this input state to the output state:

$$\begin{aligned}\hat{a}^\dagger \hat{b}^\dagger |\text{vacuum}\rangle &= (t\hat{c}^\dagger - r\hat{d}^\dagger)(r\hat{c}^\dagger + t\hat{d}^\dagger) |\text{vacuum}\rangle \\ &= (tr\hat{c}^\dagger \hat{c}^\dagger + t^2\hat{c}^\dagger \hat{d}^\dagger - r^2\hat{d}^\dagger \hat{c}^\dagger - rt\hat{d}^\dagger \hat{d}^\dagger) |\text{vacuum}\rangle .\end{aligned}\quad (2.6)$$

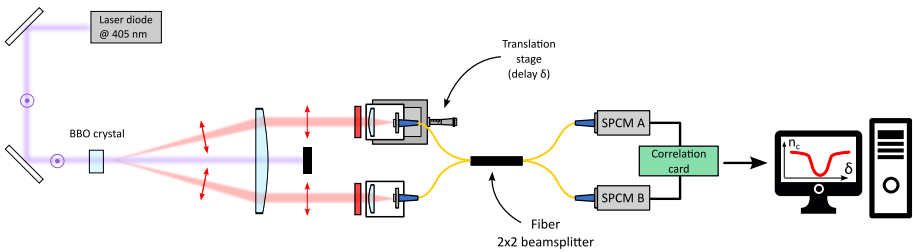
In the case of a 50/50 beam splitter, we can use the commutativity of the operators acting on the different modes of the electromagnetic field. The last expression then become:

$$\hat{a}^\dagger \hat{b}^\dagger |\text{vacuum}\rangle = \frac{1}{2}(\hat{c}^\dagger \hat{c}^\dagger - \hat{d}^\dagger \hat{d}^\dagger) |\text{vacuum}\rangle .\quad (2.7)$$

The interpretation of this equation is straightforward: at the output of the beam splitter, the states corresponding to the situation where two photons are at the output  $c$  or  $d$  are equiprobable, but the probability of having a photon on each output is zero. Therefore **we never observe any coincidence** between the detectors  $c$  and  $d$ .

## 2 Experimental realization

### 2.1 Description of the experimental setup



**Figure 2.2:** View from above. We see the laser diode, two mirrors to set the alignment, the non-linear crystal, the optics for the conjugation and two fibered collimators. The optical path of the pump beam and the photons pairs are sketched.

To observe the HOM effect, one needs to create pairs of indistinguishable photons and send them into a beam splitter (see figure 2.2). To do so, we will use

a parametric down-conversion process inside a  $\chi^{(2)}$  non-linear crystal (BBO) than can convert a pump photon at  $\lambda_p = 405 \text{ nm}$  into a pair of photons at  $2\lambda_p = 810 \text{ nm}$ . The twin photons are emitted symmetrically with respect to the pump axis, within a cone of about  $3^\circ$  of aperture. The pump beam, emitted from a laser diode, is horizontally polarized. The twin photons created in this process, are vertically polarized.

The twin photons are then injected inside two polarization-maintaining monomode fibers, thanks to a pair of fibered collimators and an optical doublet, whose focal point is set inside the crystal. An interferometric filter at  $810 \text{ nm}$  allows us to get rid of most of the stray lights. The fibered light then enters inside a module made of two waveguides that will play the role of the beam splitter. The waveguides are mounted very close to each other so that the transverse mode of the light propagating in each waveguide overlap the other one. The fibers are connected to a detection module using Avalanche PhotoDiodes (APD). A FPGA card is used to count the number of detected photons in each output. A simultaneous detection on each output is counted as a **coincidence**. A Labview code is then used to display the number of events and coincidences.

## 2.2 Single photon detection module

The single photon detection module is an exceptional tool, adapted for this kind of experiment. It is made of four fibered channels, linked to four APDs. On each channel, the detection of a photon triggers the emission of a  $25 \text{ ns}$  TTL pulse ( $0 \text{ V}$  to  $5 \text{ V}$ ). We will just use two channels out of four for this experiment, labeled *A* and *B*.

**Warning ! Those detectors are very, very expensive and would be destroyed by a strong photon flux! Always check that the interferometric filters are installed to protect the photodiodes. Wait for the teacher authorization to switch on the detectors, and make sure that the main lights are off and the door is closed.**

## 2.3 Event and Coincidence counter

The pulses and coincidences are counted by a Nucleo card collecting the pulses from the detectors, as well as the coincidence pulses generated by the output of a cascade of flip flop circuits. A coincidence pulse is generated when two pulses from the two distinct detectors reach the flip flop circuits within a coincidence window. The duration of this window is set by the time constant of RC circuits that set the reinitialization time of the flip flops.

**Q1** Why do the cables need to have the same length?

~> Make the following settings:

- Before doing anything else, switch on the power supply of the Nucleo card, start the Python program and initiate the connection to the Nucelo card.
- Switch off every lights and turn the photons counters on.
- Measure the number of dark counts. The lower this number is, the better the detectors are.
- Switch a weak light and check that the number of detected photons stays well below  $10^6$  photons/s.

## 2.4 Rate of accidental coincidences

For now, since the light sources are chaotic, the photons arriving in A and B are not correlated, and therefore the potential coincidences that you might detect are accidental. We note  $n_A$  and  $n_B$  the **counting rate** (average number of photons per second) on channels A and B,  $n_f$  the rate of accidental coincidences, and  $\tau$ , the duration of the coincidence window.

**Q2** Show that the rate of accidental coincidence is given by :  $n_f = n_A n_B \tau$ .

~> Measure the rate of accidental coincidences for the coincidence circuit.

**Q3** Calculate the duration of the coincidence window using the formula established in question **Q2**. Use this value to set the slider and the reference of the "corrected" coincidence gauge in the software.

**This measure allows one to check the behavior of the coincidence counters. Call the teacher if the results are different from the value of table ??.**

## 2.5 Pump diode

The pump diode is a 405 nm laser with about 60 mW of output power. The light emitted by the diode is linearly polarized. The wearing of security glasses is mandatory!

~ Make the following settings:

- Press the two buttons to turn on the temperature regulator of the diode. The temperature is already set to obtain the correct wavelength. Don't try to change it.
- Press the two buttons to turn on the current supply of the diode, and set the current at maximum (about 95 mA).
- Briefly check the alignment of the beams (they should be already well aligned).

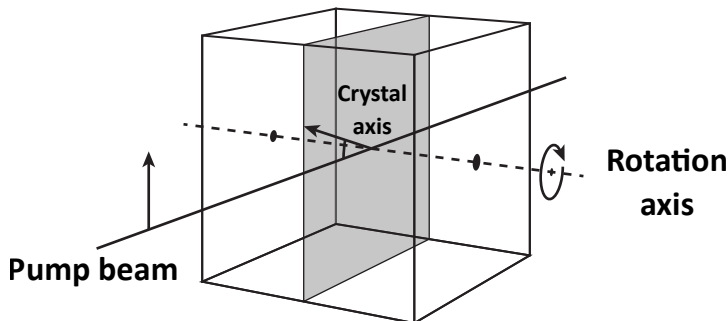
## 2.6 Parametric conversion

The photons pairs are produced by parametric down conversion in non-linear crystals  $\beta$ -BaB<sub>2</sub>O<sub>4</sub> (baryum  $\beta$ -borate, BBO for short). During the non-linear process, a photon of the 405 nm pump can be converted in a pair of twin 810 nm photons. Recall that the two photons of the pair may have slightly different energies (or wavelengths) since only their sum is set by the energy of a 405 nm pump photon (energy conservation in the process of parametric down conversion).

The BBO is a negative uniaxial birefringent crystal. We use a type I phase matching, meaning that the twin photons have the same polarization.

**Q4** Recall what are the two conditions satisfied by a non-linear process. Which one is called the phase matching condition?

The pump beam is orthogonal to the entrance plane of the crystal. The optical axis of the crystal and the axis of the pump are forming an horizontal plane (see figure 2.3).



**Figure 2.3:** Orientation of the optical axis of the crystal with respect to the pump axis.

The pump is horizontally (extraordinary) polarized and the 810 nm twin photons emitted from the crystal are vertically (ordinary) polarized. The refractive indexes  $n_o$  and  $n_e$  which characterize respectively the crystal axis and the orthogonal axis are given in table 2.1. From those indexes, one can calculate the index  $n_\theta$  seen by the pump while it propagates through the crystal:

$$1/n_\theta^2 = \sin^2(\theta)/n_o^2 + \cos^2(\theta)/n_e^2. \tag{2.8}$$

$\theta$  being the angle between the propagation axis and the crystal axis (see figure 2.3).

| wavelength (nm) | $n_o$     | $n_e$     |
|-----------------|-----------|-----------|
| 405             | 1,691 835 | 1,567 071 |
| 810             | 1,660 100 | 1,544 019 |

**Table 2.1:** Refractive index of the BBO crystal along its optical axis, at a temperature of 293 K.

**Q5** Using a spreadsheet (Excel), calculate  $n_e(405, \theta)$  as a function of  $\theta$  in the near range of  $\theta = 30^\circ$ . For what precise angle  $\theta$  is type I collinear phase tuning achieved?

If we deviate slightly from this precise  $\theta$  angle by modifying the crystal's inclination, we'll still obtain a type I, but non-collinear, phase tuning.

**Q6** The collimators are positioned in such a way that they are forming with the crystal an isosceles triangle, with a  $3^\circ$  aperture. What should be the angle  $\theta$  for the twin photons to be correctly collected by the collimators?

## 2.7 Settings of the collimators

The optical fibers are polarization-maintaining fibers. Their own axis are parallel to the horizontal and vertical axis. One of the own axis can be spotted thanks to a tip on the connector.

**Q7** Why are those polarization-maintaining fibers crucial to observe the HOM effect?

Now you need to image the area of emission of the photons pairs onto the core of each fibers, by tuning the orientation of the collimators. This is a difficult setting because the fibers' cores have a diameter of  $5\mu\text{m}$  only. A trick is to make light propagates in the other direction (from the collimators to the

crystal) by injecting a 670 nm auxiliary laser at the output of the polarization-maintaining fibers. You then have two beams coming out of the collimator. Focalise them inside the crystal and superimpose them to the pump beam. Of course you need to take the filter away from the collimator because they would cut the 670 nm light. Don not forget to put them back before switching the photon counters on again.

**Warning !** You must switch the single photon detection module off every time the fibers are disconnected, and every time the filter are taken away from the collimators. Don not forget to put them back before switching the photon counters on again. Always protect the fibers' extremity with a cap.

Even with this method, it is difficult to inject the fibers. You will have to be very cautious (and probably try several times) to set the collimators correctly. However, as soon as you manage to inject a small fraction of the twin photons, the setting becomes a lot easier, you then just have to optimize the number of coincidences by fine-tuning the position of the collimators.

**Remark** The screws of the collimators' mounts are the only elements you need to touch during this setting. If you lose all the signal in the process (and you are not able to recover it) call the teacher.

↪ Measure the coincidences rates.

**Q8** Calculate the rate of accidental coincidences.

**Q9** Do you need to account for them?

## 2.8 Observation of the HOM effect

Once the coincidence rate is high enough (about 600 coincidences/s), you are ready to observe the HOM effect. One of the collimators is mounted on a translation stage. Record the number of coincidences as you gently move the translation stage (for instance by step of  $10\mu\text{m}$ ). To lower the uncertainty of your measurements, you need to raise the number of detected coincidences, so you need to count coincidences over a longer time interval. The fluctuations of the number of coincidences  $N_c$  measured during a time  $T$  is linked to the photonic shot noise, which follows a Poisson law. It means that the standard deviation  $\sigma[n_c]$  on the coincidence rate  $n_c = N_c/T$  verifies :

$$\sigma[n_c] = \frac{\sigma(N_c)}{T} = \frac{\sqrt{N_c}}{T} = \sqrt{\frac{n_c}{T}}, \quad (2.9)$$

which leads to the relative uncertainty :

$$\frac{\sigma[n_c]}{n_c} = \frac{1}{\sqrt{N_c}} = \frac{1}{\sqrt{n_c T}}. \quad (2.10)$$

**Q10** Assuming you count an average of 100 coincidences/s, what is the standard deviation on the coincidence rate? How long should the time interval be to reduce this deviation by a factor of 10?

In practice, you will count the coincidences during a time interval of 10 or 20 s.

~ Plot the coincidence rate as a function of the position of the translation stage. Add the error bars on your graph.

**Q11** Interpret the curve.

~ What is the depth of the dip?

**Q12** Why does it not go to zero?

~ What is the the full width at half minimum of the dip?

**Q13** Compare it to the coherence length of the 810 nm photons, which is given by the the filter (10 nm width).

The width of the HOM dip is given by the energy difference of the twin photons created.

**Q14** Explain why, using the notion of indistinguishability of the photons detected.

It is now possible to add interference filters of width 5 nm.

**Q15** What width of dip should we get in this case?

↪ It is necessary to slightly change the pump wavelength and set it at 404 nm so that the twin photons are indeed transmitted by the pair of superimposed filters. To do so, you can increase the resistance of the temperature controller up to a value typically between 12,9 k $\Omega$  and 13,3 k $\Omega$ . You can use the spectrometer to check the pump wavelength.

Careful, changing the pump wavelength implies to slightly realign the collimators to collect the pairs of photons and measure coincidences. With these narrower filters and working around 808 nm the typical counts and coincidence rates are much smaller than in the previous case and require longer integration time.

↪ If you have time, add the 5 nm filters in front of the 10 nm wide filters and repeat the HOM dip measurements.

↪ Measure the new width of the HOM dip.

**Q16** Compare it with the new energy difference of the photons detected using the 5 nm width filters.



## P 3

# NV center magnetometry

### Contents

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# 1 Learning objectives

At the end of this labwork session, you'll be able to :

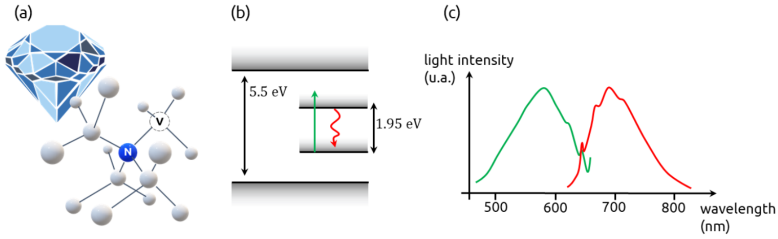
- use a method to align a light beam ("walking the beam")
- measure the RF spectrum of NV centres
- analyze the RF spectrum of NV centers under an externally applied magnetic field
- reconstruct a vectorial magnetic field from this RF spectrum
- describe a method to reconstruct weak magnetic fields such as the magnetic field of the Earth.

# 2 Introduction

An NV center is a crystalline defect of diamond. It's also known as a "colored center", because on a macroscopic scale, when these defects are present in very large numbers, they give diamond a colored hue. Since diamond is made up of a network of carbon atoms, the NV center consists of the substitution of one of these atoms by a nitrogen atom (N), directly adjacent to a vacancy (V). (see Fig. 3.1).

In the quantum world, electrons are described by a wave function. The wave functions of electrons around an atom are confined (because the electrons remain close to the nucleus). The confinement of this wave gives rise to the quantization of its energy states, in the same way that a wave (e.g. a light wave) sees its modes of propagation confined by the existence of boundary conditions, in a cavity or a waveguide.

In the case of NV centers, the disruption of the crystal lattice causes the wave function of certain electrons to be confined to the defect. It is therefore often said that an NV center behaves like an artificial atom: while crystalline diamond is a large-gap semiconductor, the defect introduces electronic energy levels within this gap.



**Figure 3.1:** Diamond NV center. (a) Schematic representation of an NV center, with a nitrogen atom (N) next to a vacancy (V). (b) Energy levels of diamond. Diamond is a wide-gap semiconductor. The presence of an NV center creates a sublevel within the gap.

The optical transition of the defect is at 637 nm. The interaction between this defect and the crystal, via phonons, causes a significant broadening of this transition. In practice, the absorption spectrum of the defect consists of a broad lobe centered around 550 nm. Fluorescence at this level undergoes a Stokes shift, and takes the form of a broad lobe centered around 700 nm, in the red part of the visible spectrum. Each lobe is around 200 nm wide.

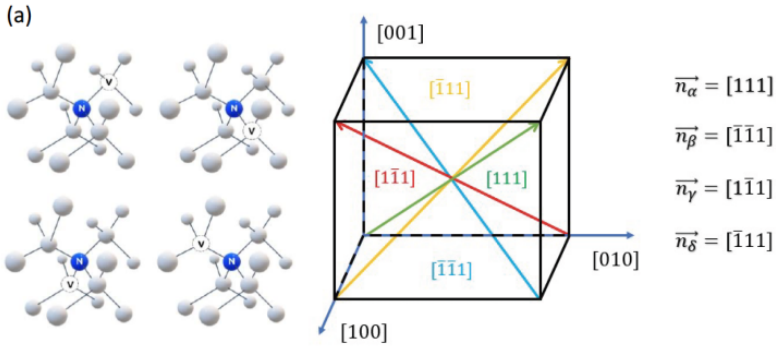
Although NV centers are naturally present in diamond, it is possible to introduce NV centers in controlled densities into diamond samples (usually synthesized).

## 2.1 Defect geometry

Diamond is a crystal with a...diamond-like structure. It is a crystal with a blende structure, in which all sites are occupied by carbon atoms. It can be described as a face-centered cubic structure in which half the tetrahedral sites are occupied, two in the lower half of the cube describing the mesh, along a small diagonal, and two in the upper half, along the other small diagonal

The structure of an NV center is inscribed in this mesh, by substitution of one of the carbon atoms by a nitrogen, and creation of a gap by replacement of one of the surrounding carbon atoms.

The natural axis of symmetry of an NV center is the axis connecting the nitrogen atom to the gap. By construction, there are therefore 4 possible orientations of an NV center in a diamond crystal, corresponding to the 4 possible vacancy positions for a given nitrogen atom. (see Fig 3.2).



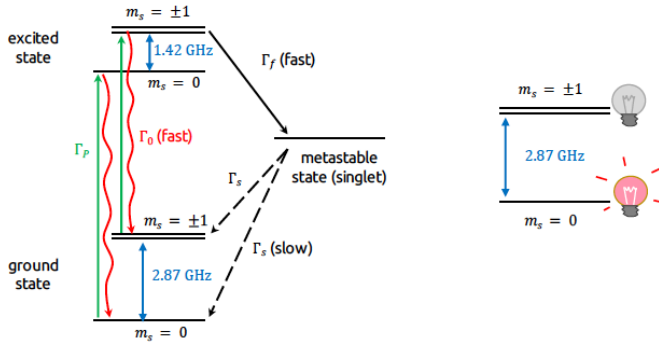
**Figure 3.2:** The four possible orientations of NV centers in a solid diamond crystal and corresponding Miller notations.

The corresponding directions can be assigned to so-called Miller indices. These are an notation system allowing to define orientations with respect to the crystalline axis if the system.

Here, we define the known orientation of the diamond sample with respect to the lab reference frame as  $(X, Y, Z)$ . You can find at the end of this textbook an appendix explaining in more details the relation between the Miller indices and the axes  $(X, Y, Z)$ .

### 3 Energy levels and spin

An NV center has a total spin  $S = 1$ . The fundamental level and the excited level of the optical transition of interest are therefore spin triplets: they are characterized by the quantum number  $m_S$ , which can take on the values 0,1, depending on the projection of the spin on the quantization axis (which is therefore the axis of the NV center). (see Fig 3.3).



**Figure 3.3:** Energy level structure of an NV center, comprising a triplet of ground states and a triplet of excited states

The energy degeneracy between the  $|m_S = \pm 1\rangle$  and  $|m_S = 0\rangle$  levels is partially overcome by spin-spin interaction. These two sets of levels are separated by an energy whose corresponding frequency is  $D \approx 2.87\text{GHz}$  for the fundamental triplet and  $D' \approx 1.42\text{GHz}$  for the excited triplet.

It is therefore possible to use a laser to induce an optical transition from one of the fundamental levels to the excited state of the same spin.

Moreover, one can use a radio-frequency antenna tuned at 2,87 GHz to induce transitions within the levels  $m_s = 0$  and  $m_s = \pm 1$ .

**A crucial point in the energy level structure of an NV center is the presence of a metastable singlet state.** The excited state  $|m_S = 0\rangle$  is not coupled to this metastable state and will always decay to the corresponding ground state by emitting a photon. For excited states  $|m_S = \pm 1\rangle$ , however, there is a probability of non-radiative de-excitation from the levels to the metastable state, i.e. without photon emission.

In other words, when the NV center is placed in the  $|m_S = \pm 1\rangle$  states, it emits less fluorescence as it de-excites: this property is at the heart of the experiments that will be carried out during the course of the labwork.

### 3.1 Zeeman effect

The energy levels of the NV center are given by the Hamiltonian of the system. In the  $(x_{\text{NV}}, y_{\text{NV}}, z_{\text{NV}})$  basis related to the NV center, with the  $z_{\text{NV}}$  axis aligned with the NV center, this Hamiltonian is written :

$$\hat{H} = hD\hat{S}_{z_{NV}}^2 + hE(\hat{S}_{x_{NV}}^2 - \hat{S}_{y_{NV}}^2) + \Gamma(\mathbf{B} \cdot \hat{\mathbf{S}}) \quad (3.1)$$

where

- $D \approx 2.87 \text{ GHz}$  is the zero-field separation factor of the fundamental levels, related to the spin-spin interaction.
- $E$  is a splitting factor related to imperfections in the crystal lattice,
- $\Gamma \approx 28 \text{ MHz.mT}^{-1}$  is the gyromagnetic factor. More precisely, one writes  $\Gamma = g\mu_B$ , with  $g$  the Landé factor and  $\mu_B$  the Bohr's magneton associated with the  $\mathbf{S}$  spin of the NV centers.
- $\mathbf{B}$  represents the external magnetic field that can be applied.
- $\mathbf{S}$  represents the spin operator and its components, the Pauli spin matrices;

Under some approximations, the Hamiltonian can be simplified to contain only the contribution of the magnetic field aligned with the center NV :

$$\hat{H} \approx hD\hat{S}_{z_{NV}}^2 + \Gamma(B_{z_{NV}}S_{z_{NV}}) + hE(\hat{S}_{x_{NV}}^2 - \hat{S}_{y_{NV}}^2) \quad (3.2)$$

This Hamiltonian explains the 2,87 GHz splitting between level  $m_s = 0$  and the levels  $m_s = \pm 1$  degeneracy lift of the  $ket m \pm 1$  levels. Moreover, the degeneracy between levels  $m_s = \pm 1$  can be lifted by two contributions :

- at zero field, a first contribution arises from the deformations of the crystal lattice (term in  $E$ )
- in addition, a second contribution to the splitting arises from the amplitude of the projection of the magnetic field onto the axis of the NV center: this is the Zeeman effect, modeled by the term  $(B_{z_{NV}}S_{z_{NV}})$ .

The frequencies of both levels  $m_S = -1$  and  $m_S = +1$  are given by :

$$f_{\pm} = D \pm \sqrt{E^2 + (\Gamma B_{z_{NV}})^2} \quad (3.3)$$

For intense fields, the Zeeman splitting is therefore given by :

$$\Delta f_{\text{Zeeman}} = f_+ - f_- \approx 2\Gamma B_{z_{NV}} \approx 56 \cdot \text{MHz/mT} \cdot B_{z_{NV}} \quad (3.4)$$

### 3.2 Preliminary questions

The session consists in carrying out *Optically Detected Magnetic Resonance* (or ODMR) measurements. The protocol is as follows: the fluorescence of an NV center illuminated by a laser is detected. The NV center itself is placed on an RF antenna. The fluorescence intensity is observed by switching on the RF antenna and performing a frequency sweep around the transition between the  $|m_S = 0\rangle$  and  $|m_S = \pm 1\rangle$  states of the fundamental triplet.

**P1** Schematically represent the ODMR spectrum of a single NV center at zero field ( $B = 0$ ), then at non-zero field.

**P2** How does this ODMR spectrum measure the amplitude of a magnetic field component?

**A diamond crystal can contain numerous NV centers, in 4 possible orientations. A fluorescence experiment thus simultaneously probes 4 different populations of NV centers, each associated with a different projection of the same magnetic field on the center's orientation. An ODMR plot can therefore display up to 4 pairs of dips, thus with exhibiting 4 different Zeeman splittings.**

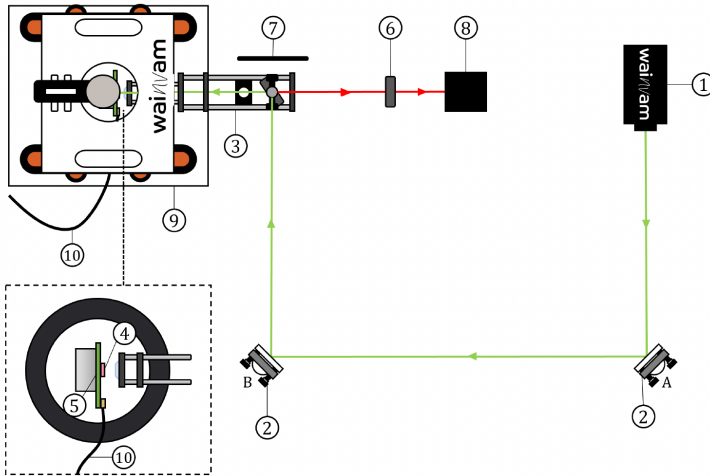
**P3** Schematically represent the ODMR spectrum of a diamond crystal containing numerous centers at zero field, then at non-zero field.

## 4 Optical setup

### WARNING !!!

In this TP, the maximum power of the 532 nm laser is 140 mW, i.e. over 100 times the eye's permanent damage threshold.

Please wear safety goggles when the laser is in operation.  
For any changes to be performed on the optical table, lower the laser power below 1 mW, then open the boxes before adjusting.



**Figure 3.4:** Setup schematic 1 - Laser source. 2 - Alignment mirrors. 3 - Caged system comprising irises, dichroic mirror and aspherical focusing lens. 4 - NV-doped diamond sample. 5 - RF antenna, sample support. 6 - High-pass filter ( $\lambda$ ). 8 - Detector. 9 and 10 - Coils and their power supplies.

The optical set-up (see Figure 3.4) consists of a 532 nm laser, sent by two mirrors to a dichroic mirror, towards an asphere lens focusing the beam on a small diamond sample rich in NV centers, placed on an RF antenna, and in the middle of three pairs of coils allowing magnetic fields to be applied along the X, Y and Z axes (linked to the laboratory reference frame). The red fluorescence of the NV centers follows the opposite path, but is transmitted by the dichroic mirror, towards a photodiode.

The laser excitation power level and the photodiode detection signal can be

controlled via the WAINTEACH control software.

## 5 Experiments

### 5.1 Aligning the beam ("beam walking")

↪ Switch on the laser and set the drive current up to few mA.

Before starting measurements, fine-tuning the alignment of the laser on the sample might be necessary. To do this, one may adjust the micrometric screws on the two mirrors, so that the beam passes through two diaphragms, whose straight line joining the two centers defines the optical axis of the setup.

As the rest of the set-up has already been adjusted, it is not necessary (and not recommended) to touch other elements of the set-up. If you feel the need (or curiosity!) to do so, please call the teacher.

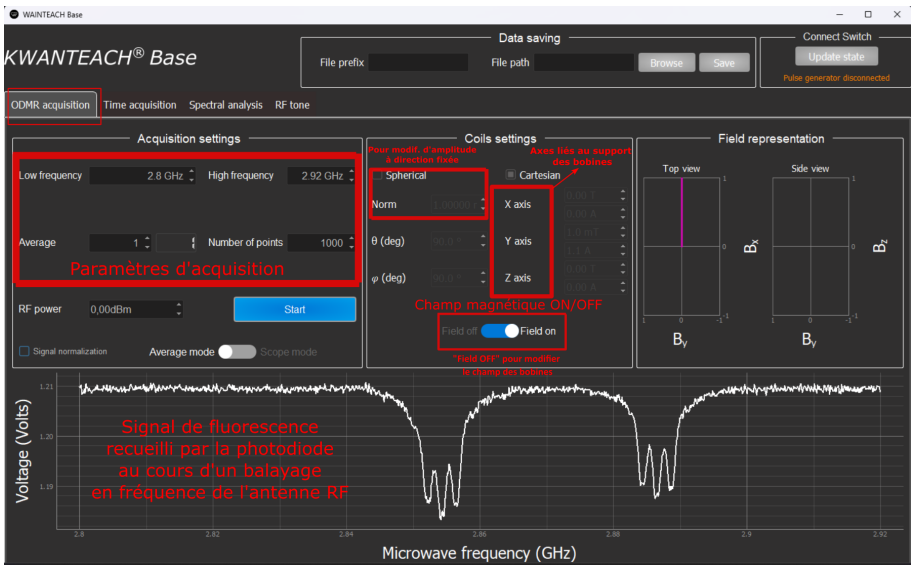
→ You can optimize further the mirror orientation by optimizing the collection of the fluorescence signal by the photodiode. To do so, connect the output of the photodiode to the oscilloscope.

→ When the laser is adequately focused onto the crystal, one can easily notice a strong orange fluorescent hue, even easier to wear with the safety goggles on.

### 5.2 Zero field measurement

The aim of this first experiment is to carry out an ODMR measurement at zero field - without applying a magnetic field to the coils.

- ⇒ With the boxes closed, turn up the laser power in a range between 20 mW and 80 mW.
- ⇒ Start the software **kwanteach-sw** using the shortcut icon on the desktop. In the software window, click the "load wainteach" button.
- ⇒ Display the "ODMR Acquisition" tab in the Kwanteach software (voir Figure 3.5).
- ⇒ Record a first ODMR spectrum, for example between 2,8 GHz and 2,94 GHz, with RF power set at 0 dBm.



**Figure 3.5:** Interface du logiciel dans l'onglet "ODMR acquisition".

⇒ Save this acquisition (and subsequent ones) in the same folder, using the software's "save" function.

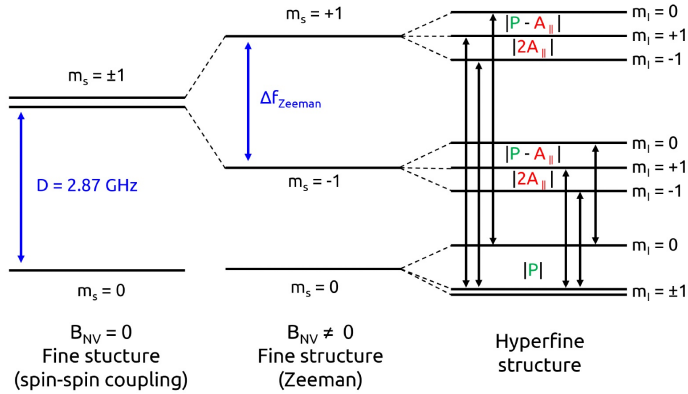
↪ Click on the recorded plots to show a cursor that will enable you to precisely display the corresponding frequency.

**Q1** Note the center frequency corresponding to fluorescence drop. How do you explain this drop? What transition between levels in the NV center does this center frequency correspond to?

**Q2** If you look carefully, you should see two peaks within the central peak. Which Hamiltonian term explains the physical origin of these two peaks? Comment.

### 5.3 Measurement in an unspecified magnetic field

⇒ Bring the horseshoe magnet close to the sample, from the top of the coils. Repeat the spectrum measurement.

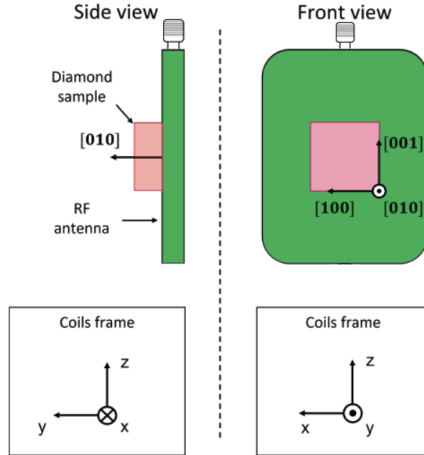


**Figure 3.6:** Details of the energy level structure of a single NV center, down to its hyperfine structure..

**Q3** What do you observe? Describe the changes observed and explain qualitatively why the shape of the ODMR spectrum has changed.

**Q4** Use the hyperfine level structure described on Figure 3.6), and comment the number of observable peaks in this non-specified magnetic field.

## 5.4 Influence of the magnetic field orientation



**Figure 3.7:** Diagram of the sample orientation within the coils applying the magnetic field. The diamond is cut perpendicular to the orientation  $[010]$  - each of the 3 axes of the coils corresponds to one of the crystalline axes of the diamond lattice.

The diamond sample is oriented along the axes shown in figure 3.7. The aim of this section is to study the effect of field orientation on the ODMR spectra.

The measurement bench allows a magnetic field with a controlled orientation to be applied using three pairs of coils. **Helmholtz coils** are used here: each pair consists of two identical circular coils with radius  $R$ , positioned parallel to each other and facing one another at a distance  $d$  equal to their radius. The advantage of this commonly used configuration is that it produces a highly uniform magnetic field at the center of the setup, along the axis connecting the coils, and over a region comparable in size to  $R$ . By combining three pairs of coils aligned along the  $X$ ,  $Y$ , and  $Z$  axes of the laboratory frame, one can exploit the linearity of Maxwell's equations to sum the contributions of each pair and generate a magnetic field with an arbitrary orientation at the center of the system.

⇒ A gaussmeter is used to measure magnetic fields. Due to a significant offset, differential measurements must be performed to obtain accurate values along a given axis. For example, to measure the component of the field along the  $OXOX$  axis, the gaussmeter should be oriented first along  $+X$ , then along

$X$ , by rotating the Hall probe by 180 deg.

**Q5** Measure the magnetic field in the room along the three orientations  $X, Y, Z$ . Estimate the Earth's magnetic field. A compass can be used to verify the direction of magnetic North.

Repeat the measurement by placing the gaussmeter at the center of the coils, with the system powered off. Then, apply several current values to the coils and verify that the readings provided by the gaussmeter are consistent with those displayed by the software.

The diamond sample is oriented according to the axes shown in Figure 3.7. The aim of this section is, on the one hand, to study the effect of the magnetic field orientation on the ODMR spectra.

**Q6** Recall the possible orientations of the NV centers in the crystal axis system. Associate each of these orientations with a unit vector in the coil axis system  $X, Y, Z$ .

⇒ Apply a magnetic field of approximately 2 mT along the  $Y$  axis, the normal to the sample.

**Q7** What is the projection of this field onto the four possible NV center orientations? Use your answer to comment on the shape of the resulting ODMR spectrum.

**Q8** If the magnetic field had a different projection onto each of the four possible NV center orientations, what would be observed? Try to identify a field orientation that would result in the maximum number of peaks.

**Q9** Align the magnetic field along the direction  $\beta = \frac{1}{\sqrt{3}}(1, 1, 1)$ . Why do two pairs of peaks appear? Why do these two pairs of peaks have very different amplitudes? Identify each of these peaks.

## 5.5 Measuring the gyromagnetic ratio

The aim of this section is to study the influence of the field norm on Zeeman splitting, for a class of NV centers with a given orientation.

- ⇒ Keep the magnetic field oriented along direction  $\beta$ .
- ⇒ Once the direction of the magnetic field is set, use the "Spherical" tab in the "Coils settings" panel to vary the magnetic field strength from 0 to 2.5 mT in steps of 0.5 mT.
- ⇒ For each applied field value, and after averaging over 5 consecutive sweeps, record the frequency position of the central peak of each of the two observed triplets.

**Q10** Use your measurement to plot the evolution of the Zeeman splitting with respect to the amplitude of the applied magnetic field. Deduce the gyromagnetic factor  $\Gamma$ .

## 5.6 Measuring the Earth magnetic field

In practice, the magnetometer never rests in a rigorously zero magnetic field, even when the coils are switched off: it rests in the Earth's magnetic field (and other parasitic fields).

**Q11** Using your previous gaussmeter measurements, provide an estimate of the expected Zeeman splitting amplitude for an NV center oriented vertically.

The goal of this section is to determine the magnitude and orientation of the Earth's magnetic field. More generally, the method you will follow can be used to measure and characterize weak and unknown magnetic fields at very small scales.

### Measurement guidelines

Measuring a magnetic field with a magnetometer is based, on the one hand, on the idea that the Zeeman splitting of a pair of peaks is linked to the norm of the projection of the magnetic field on the direction associated with an NV center, and on the other hand, on the exploitation of the 4 possible NV center orientations to obtain 4 different projections of the same magnetic field vector on different directions.

The magnetic field of the coils is therefore added to that of the earth's magnetic field to obtain adequate ODMR spectra. A complete procedure to reconstruct the ambient magnetic field from a sequence of ODMR measurements is detailed in the appendix to this session's textbook.

### Experimental protocole

- ⇒ Record the frequency position of the central peak of the two outermost triplets;
- ⇒ Acquire a second spectrum by *reversing the direction of the applied magnetic field*. The total field experienced by the NV centers becomes  $\vec{B}_{tot}^- = \vec{B}_{Earth} - \vec{B}_\alpha$ ;
- ⇒ Again, record the frequency position of the central peak of the two outermost triplets;
- ⇒ Repeat these measurements by applying a magnetic field along the other crystalline orientations  $\beta$ ,  $\gamma$ , and  $\delta$ , each time in both opposite directions.

**Q12** Use the Excel sheet available on the computer and report your measurement results enabling you to calculate the projection of the ambient magnetic field onto each one of the four different crystalline orientations. The table then provides a way to compute the magnetic field components in the reference frame of the lab,  $X$ ,  $Y$  and  $Z$ . Comment on the main sources of uncertainties of your setup and suggest solutions to reduce them as much as possible.

## Appendix: magnetic field reconstruction

### Expression of Zeeman splitting as a function of magnetic field projections

To reconstruct the magnetic field from ODMR spectra, each ODMR line must first be assigned an NV center class and orientation. To do this, we need to go back to the projection of the magnetic field onto each orientation.

The two split frequencies  $f_{i\pm}$  associated with a given NV center orientation  $n_i$  (with  $i = \alpha, \beta, \gamma, \delta$ ) are expressed as :

$$f_{i,\pm} = f_0 \pm \gamma |\mathbf{B}_{tot}| \cos \theta_i \quad (3.5)$$

with  $\theta_i$  the angle between the magnetic field vector and the crystal orientation vector.

In the approximation where the ambient field is much weaker than that applied by the coil, this angle is given by :

$$\frac{\mathbf{B}_{\text{coil}} \cdot \mathbf{n}_i}{|\mathbf{B}_{\text{coil}}||\mathbf{n}_i|} \approx \cos \theta_i \quad (3.6)$$

The sign matters and the cosine can be negative, so that  $f_{i+} > f_{i-}$  if  $\cos \theta_i > 0$ , and  $f_{i+} < f_{i-}$  if  $\cos \theta_i < 0$ . The Zeeman splitting, which can be positive or negative, is then expressed as :

$$\Delta f_i = |f_{i+} - f_{i-}| \times \text{sign}(\cos(\theta_i)) \quad (3.7)$$

## Measuring a magnetic field component

In the labwork, two consecutive spectra are measured by reversing the direction of the magnetic field, resulting in two splitting measurements.

$$\mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{Earth}} + \mathbf{B}_{\text{coil}} \longrightarrow \Delta f_i^+$$

$$\mathbf{B}_{\text{tot}} = \mathbf{B}_{\text{Earth}} - \mathbf{B}_{\text{coil}} \longrightarrow \Delta f_i^-$$

Using the projection angle  $\theta_i$ , we can write :

$$\Delta f_i^+ = 2\gamma B_{\text{Earth}} \times \cos \theta_i + 2\gamma B_{\text{coil}} \times \cos \theta_i \quad (3.8)$$

and likewise,

$$\Delta f_i^- = 2\gamma B_{\text{Earth}} \times \cos \theta_i - 2\gamma B_{\text{coil}} \times \cos \theta_i \quad (3.9)$$

Combining the two expressions gives :

$$\frac{\Delta f_i^+ + \Delta f_i^-}{4} = B_{\text{Earth}} \times \cos \theta_i = B_{\text{Earth},i} \quad (3.10)$$

i.e. the measurement of the projection of the value of the earth's magnetic field on the direction  $\mathbf{n}_i$ .

## Crystal orientations and field reconstruction

The orientations of the various NV center classes can be linked to the axes of the laboratory's frame of reference  $\mathbf{n}_\alpha, \mathbf{n}_\beta, \mathbf{n}_\delta, \mathbf{n}_\gamma$

$$[111] \longrightarrow \mathbf{n}_\alpha = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$[\bar{1}\bar{1}1] \longrightarrow \mathbf{n}_\beta = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

$$[\bar{1}11] \longrightarrow \mathbf{n}_\gamma = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$[1\bar{1}1] \longrightarrow \mathbf{n}_\delta = \frac{1}{\sqrt{3}} \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

This translates directly into a system of equations linking the Cartesian components of the field to those projected onto the crystal orientations:

$$\begin{aligned} \sqrt{3} \times B_{\text{Earth},\alpha} &= +B_{\text{Earth},x} + B_{\text{Earth},y} + B_{\text{Earth},z} \\ \sqrt{3} \times B_{\text{Earth},\beta} &= -B_{\text{Earth},x} - B_{\text{Earth},y} + B_{\text{Earth},z} \\ \sqrt{3} \times B_{\text{Earth},\gamma} &= +B_{\text{Earth},x} - B_{\text{Earth},y} + B_{\text{Earth},z} \\ \sqrt{3} \times B_{\text{Earth},\delta} &= -B_{\text{Earth},x} + B_{\text{Earth},y} + B_{\text{Earth},z} \end{aligned}$$

which gives :

$$\begin{aligned} B_{\text{Earth},x} &= \frac{\sqrt{3}}{4} (B_{\text{Earth},\alpha} - B_{\text{Earth},\beta} + B_{\text{Earth},\gamma} - B_{\text{Earth},\delta}) \\ B_{\text{Earth},y} &= \frac{\sqrt{3}}{4} (B_{\text{Earth},\alpha} - B_{\text{Earth},\beta} - B_{\text{Earth},\gamma} + B_{\text{Earth},\delta}) \\ B_{\text{Earth},z} &= \frac{\sqrt{3}}{4} (B_{\text{Earth},\alpha} + B_{\text{Earth},\beta} + B_{\text{Earth},\gamma} + B_{\text{Earth},\delta}) \end{aligned}$$



## P 4

# Saturated absorption Sub-Doppler spectroscopy

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At room temperature, the main cause of the broadening of atomic transitions in a gas is the Doppler effect. Saturated absorption spectroscopy, developed in the 1970s, makes it possible to overcome this broadening and resolve the hyperfine structure of atomic transitions. The aim of the practical work is to carry out saturated absorption spectroscopy of the D1 line of rubidium in order to observe its hyperfine structure. The source used is a laser diode tunable in wavelength around 795 nm.

## 1 Preliminary study

### 1.1 Doppler broadening

We consider an atomic gas of two-level atoms and denote  $\nu_0$  the frequency of the atomic transition. Doppler broadening, which is largely predominant at room temperature, results from the dispersion of the velocities of the atoms in the gas. The gas is illuminated by a laser beam of frequency  $\nu$  propagating along the axis  $Oz$  in the positive direction. Among the atoms, only those whose  $V_z$  projection of the velocity along  $Oz$  satisfies the relation

$$\nu_0 = \nu (1 - V_z/c) \quad (4.1)$$

are in resonance with the wave and can absorb and re-emit light ( $c$  is the speed of light). Thus, if we sweep the laser frequency, **we obtain an absorption line profile that reflects the distribution of atomic velocities in the direction of the laser beam.**

**P1** Quickly explain the demonstration of the formula (4.1).

Statistical physics shows that the velocity distribution of atoms in a perfect gas follows the Maxwell-Boltzmann law:

$$f(V_z) = \left( \frac{m}{2\pi k_B T} \right)^{1/2} \exp \left( \frac{-m V_z^2}{2k_B T} \right). \quad (4.2)$$

In this equation,  $V_z$  is the velocity along the  $z$  axis,  $T$  is the temperature of the gas,  $m$  is the mass of the atoms ( $1.41 \cdot 10^{-25}$  kg for rubidium) and  $k_B$  is Boltzmann's constant ( $1.38 \cdot 10^{-23}$  J/K).

**P2** Show that the width at half-height of the line broadened by the Doppler effect is :

$$\Delta\nu = \sqrt{8 \ln 2} \sqrt{\frac{k_B T}{m}} \frac{\nu_0}{c}. \quad (4.3)$$

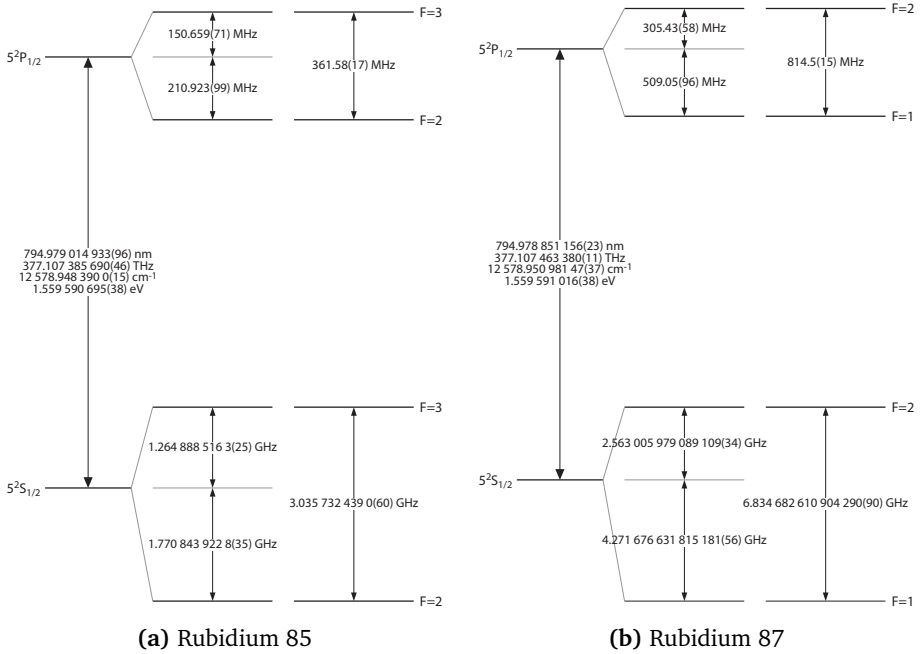
**P3** Use the formula 4.3 to calculate the Doppler broadening at  $T = 20^\circ$  of the rubidium D1 line (centred at 795 nm).

## 1.2 Structure of the rubidium D1 line

The rubidium cell you are going to use contains the two stable isotopes of the atomic species in the proportions of their natural abundance: 72% rubidium 85 and 28% rubidium 87. The wavelength of study (795 nm) corresponds to the transition from the fundamental level  $5S_{1/2}$  to the excited level  $5P_{1/2}$ . The hyperfine structure is detailed in figure 4.1 for both isotopes.

**P4** How many lines should we theoretically observe? Arrange these lines in ascending order of resonance frequency.

**P5** Taking Doppler broadening into account, which lines can be resolved with a simple absorption spectrum? At what temperature would we need to go down to resolve the entire hyperfine structure? Knowing that the natural width of the excited levels is 6 MHz, at what temperature would we have to go down to be able to measure it? How can this be achieved?



**Figure 4.1:** Hyperfine structure of the D1 line of the two rubidium isotopes. The figure is taken from: Daniel A. Steck, *Rubidium D Line Data*, available online at <http://steck.us/alkalidata>.

### 1.3 Saturation of absorption

Let's now return to a two-level gas of atoms. In general terms, the absorption  $\mathcal{A}$  (in  $\text{cm}^{-1}$ ) of a laser beam of frequency equal to the atomic transition frequency is given by the effective resonance scattering cross-section  $\sigma_0$  (in  $\text{cm}^2$ ) weighted by the difference between the densities of atoms in the fundamental level,  $n_1$ , and in the excited level,  $n_2$  (in  $\text{cm}^{-3}$ ). This difference in density depends on the intensity  $I$  (in  $\text{W} \cdot \text{cm}^{-2}$ ) of the laser beam and therefore on the absorption  $\mathcal{A}$ . This process of circular dependence ( $\mathcal{A}$  depends on  $n_1 - n_2$ , which depends on  $I$ , which depends on  $\mathcal{A}$ ) gives rise to the phenomenon of absorption saturation. In concrete terms, we show that:

$$\mathcal{A} = \sigma_0 \times (n_1 - n_2) \quad \text{avec} \quad n_1 - n_2 = \frac{n_t}{1 + I/I_{\text{sat}}} , \quad (4.4)$$

where  $n_t$  is the total density of atoms and  $I_{\text{sat}}$  is an intensity characteristic of the transition, called the **saturation intensity**, which is  $1.6 \text{ mW/cm}^2$  for the atomic transition under consideration. The ratio  $s = I/I_{\text{sat}}$ , which is used in

the equation 4.4, is called the **saturation parameter**.

**P6** Make a qualitative comparison of the absorption in the limiting cases  $s = I/I_{\text{sat}} \ll 1$  and  $s = I/I_{\text{sat}} \gg 1$ . What about absorption in the case of high intensity ( $s \gg 1$ ). **This phenomenon is called absorption saturation.**

**P7** Give the limit of the populations  $n_1$  and  $n_2$  in the case of strong saturation. Give an interpretation of absorption saturation by considering the evolution of the spontaneous emission rate as a function of intensity.

**P8** Is it possible to invert the populations on a 2-level system? Explain why.

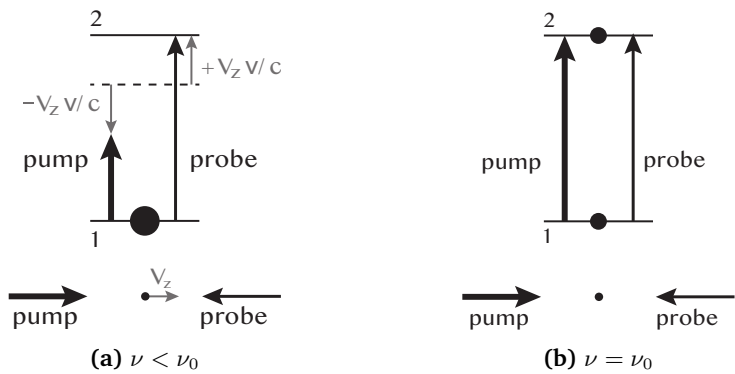
## 1.4 Application to sub-Doppler spectroscopy (pump-probe method)

Consider two laser beams of the same frequency  $\nu$  and counter-propagating (superimposed but propagating in opposite directions). The first beam, called the pump beam, propagates in the positive direction of the  $Oz$  axis and has sufficient intensity to saturate the absorption on the transition in question. The second beam, called the probe beam, propagates in the negative direction of the  $Oz$  axis and has too low an intensity to saturate the transition on its own. We are interested here in the absorption of the probe beam in the presence of the pump beam when their frequency is varied around the frequency of the atomic transition.

According to equation 4.1, for a given frequency  $\nu$ , the atoms which absorb the pump are those for which  $V_z = -c(1 - \nu/\nu_0)$  and the atoms which absorb the probe are those for which  $V_z = +c(1 - \nu/\nu_0)$ . There are then two cases, represented in figure 4.2 :

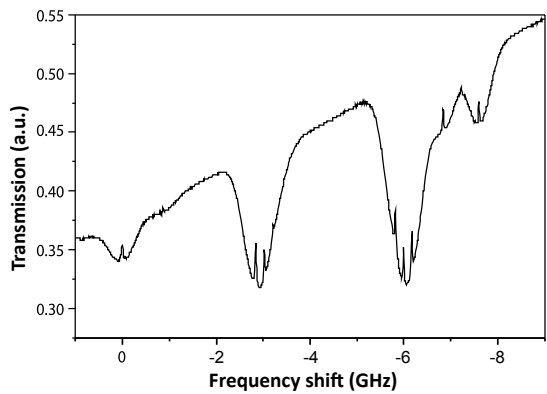
- if  $\nu \neq \nu_0$ , the atoms which absorb the pump and those which absorb the probe belong to different velocity classes. In this case, the presence of the pump has no effect on the absorption of the probe;
- if  $\nu = \nu_0$ , the same atoms absorb the pump and the probe, i.e. atoms with zero velocity along  $z$  ( $V_z = 0$ ). As the pump is sufficiently intense to saturate the absorption, the probe is absorbed very little (absorption is reduced) at this precise frequency  $\nu = \nu_0$ .

The absorption line profile of the probe therefore has a dual structure, with a narrow peak centred on the frequency of the atomic transition superimposed on the profile broadened by the Doppler effect. As you will see in the course of the labwork, the resolution offered by pump-probe spectroscopy is such that



**Figure 4.2:** Influence of the Doppler effect in the pump-probe configuration.

it can reveal the hyperfine structure of the atomic transition. Each transition then appears as a distinct peak in the Doppler profile, as shown in Figure 4.3.



**Figure 4.3:** Absorption spectrum around 795 nm obtained by the pump-probe method. The lines of the hyperfine structure appear as narrow peaks against a background broadened by the Doppler effect. The figure is adapted from: Svenja A. Knappe *et al.*, *Microfabricated saturated absorption laser spectrometer*, *Optics Express* **15**, 6293–6299 (2007).

**P9** What is the intrinsic width of the saturated absorption peak? What are the reasons why this peak might appear wider?

1.5 Laser Diode Characteristics

The tunable laser used here is a DBR795PN laser diode, typically employed to address the D1 line of rubidium. The letters DBR stand for "Distributed Bragg Reflector": a Bragg grating is used as a reflector at one end of the gain medium. The emission frequency of the diode depends on two parameters: the equilibrium temperature  $T_{eq}$  of the junction and the injection current  $i_i$  of the diode. The emission frequency decreases with both the regulated temperature  $T_{eq}$  and the injection current  $i_i$  (which slightly heats the medium), mainly due to the increase in the refractive index of the epitaxial layers with temperature. The laser "redshifts" as the current increases. In this lab session, we will vary the emission frequency using only the injection current. The full specifications of the laser diode are provided in Table 4.1.

|                                      |                |
|--------------------------------------|----------------|
| Power                                | 40 mW          |
| Wavelength                           | 795 nm         |
| Current tuning of the wavelength     | +0, 0014 nm/mA |
| Temperature tuning of the wavelength | +0, 06 nm/K    |
| Spectral width                       | 0, 25 nm       |

**Table 4.1:** Characteristics of the laser diode model DBR795PN used in this lab session. Complete specifications are available online on [thorlabs.com](http://thorlabs.com)

**P10** Compare the spectral width of the laser with the intrinsic linewidth of the transition. Which one of the two will set the observed width after suppression of the Doppler broadening ?

2 Experimental realization

**Precautions for using the laser diode** Never disconnect the cables connecting the laser diode to its power supply. **The maximum power delivered by the diode is of the order of 30 to 50 mW, which is enough to cause irreversible eye damage in direct vision.** As the eye is not very sensitive to this wavelength, the beam can reach you without you noticing. So be careful not to put your eyes at bench level. Remove or hide any reflective objects that may be at beam level (watches, bracelets, rings, chains, bracelets, belt buckles, etc.). Check that all optical elements are firmly attached to the board when the diode is transmitting. Finally, use the laser safety glasses provided at the entrance to the room.

**General precautions** All the optical elements used are fragile, expensive and difficult to obtain (particularly the cell containing the rubidium). You will be making difficult adjustments, in the dark, and with a near-infrared viewfinder (which is not very convenient). So move the elements carefully, clamp the feet carefully after each modification and don't leave anything lying around the edge of the board.

## 2.1 Laser diode

The tunable laser used here is a "DBR" (Distributed Bragg Reflector) laser diode, which provides single-mode operation at a very fine instantaneous laser linewidth (around MHz, i.e. around  $2 \cdot 10^{-6}$  nm!!). However, fluctuations in current and temperature will introduce fluctuations in this frequency and cause it to broaden. In practice, the actual frequency linewidth is therefore higher, of the order of a few tens of MHz.

The laser diode package includes an optical fiber to prevent perturbing backward light reflection into the cavity <sup>1</sup>, and the light is directly injected into a single mode polarizing maintaining optical fiber.

The laser emission frequency is a function of two parameters: the equilibrium temperature  $T_{eq}$  of the junction and the intensity  $i$  of the diode injection current. The emission frequency decreases both with  $T_{eq}$  and with  $i$  (the wavelength increases with the current). In this labwork, we will vary the emission frequency using the current alone. The full characteristics of the laser diode used are detailed in the document at your disposal.

**Temperature controller** The junction equilibrium temperature is measured by a thermistor. The set temperature is fixed by a resistance value. Its value is preset so that the entire spectrum around line D1 can be observed by sweeping the current alone. **In principle, you will not need to change this setting.** If, however, you need to change the set temperature, to move away from a mode jump for example, be careful to do so only in very small steps. A sudden change would take you too far away from resonance and it would be difficult to find it again afterwards.

**Current supply** By default, set the current to around 120 mA so that you can view the beam using the infra-red card (or simply on a piece of paper).

---

<sup>1</sup>Laser diodes are very sensitive to reflections (even very weak ones) from the beam towards the cavity, which can introduce significant fluctuations in the emission length. It is therefore necessary to use an optical isolator. This blocks the reverse return of light by using a Faraday rotator placed between two polarisers. The Faraday rotator used rotates a  $45^\circ$  rectilinear polarisation in the same direction regardless of the direction in which the light travels, so that if the polarisers are oriented to allow light propagating in one direction to pass, they will necessarily block light propagating in the other direction.

**Q1** If we want to observe all the transitions of the D1 line of rubidium for both isotopes, over what range should the current in the laser diode be varied? (To answer this, you will need to find the relevant information in the provided document...)

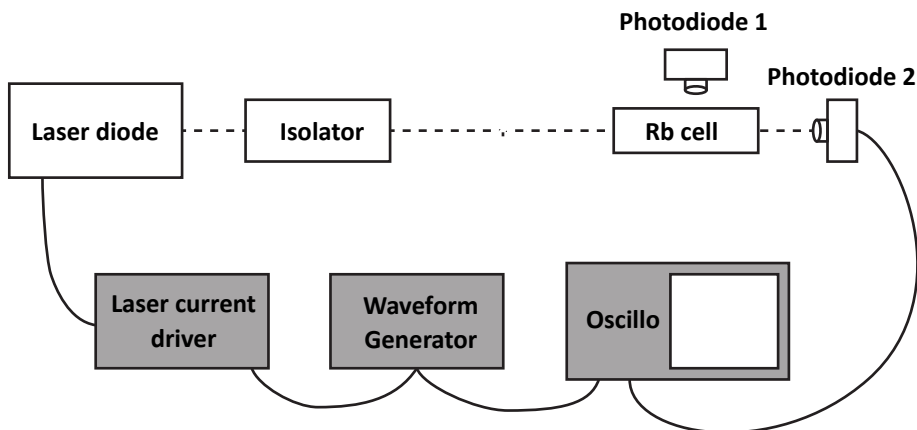
**Q2** Compare the spectral linewidth of the laser with the natural linewidth of the atomic transition. Which of these two linewidths will determine the width of the absorption lines once the Doppler effect is eliminated?

## 2.2 Fluorescence and absorption spectra

~ Place the cell in the path of the laser beam and adjust the observation camera so that the inside of the cell is clearly visible.

~ In total darkness, slowly vary the value of the current. In this way, you should be able to observe the fluorescence of the rubidium in the D1 line using the camera: the path of the laser beam shines in the cell for a precise value of current!

~ Once you have obtained fluorescence, complete the set-up described in figure 4.4. To visualise the lines of the two isotopes contained in the cell, sweep the diode's emission frequency by modulating the driving current with a triangular signal.



**Figure 4.4:** Schematic of the setup when measuring directly fluorescence and absorption.

↪ Switch on the GBF and set it to produce a low-amplitude triangular signal at a frequency of around 10 Hz. Display the GBF signal on one of the oscilloscope channels.

↪ Place one of the two photodiodes carefully against the cell (without contact), perpendicular to the laser beam, to capture part of the fluorescence signal. Monitor the photodiode signal on the other channel of the oscilloscope (set the adjustable gain to maximum) .

↪ Modify the amplitude of the modulation applied to see the different fluorescence lines expected. Improve the signal obtained (by averaging if necessary).

↪ Save the displayed signals with an USB stick (or take a photo of the screen, after you've turned on "stop").

**Q3** Identify the different lines on the oscillogram and briefly explain how you did this.

**Q4** Calibrate the oscillogram using the largest gap between two lines as a reference. Measure and check the spacing and relative positions of the different lines. Check that the width of the lines corresponds to that expected from the Doppler effect (or at least is of the right order of magnitude).

↪ Place the second photodiode behind the cell and display the absorption spectrum on the oscilloscope channel previously occupied by the GBF (the GBF is moved to the synchro input). **Caution: adjust the gain to make sure you don't saturate the signal coming out of the photodiode, which happens very quickly!**

↪ Observe the fluorescence and absorption spectra simultaneously.

↪ Observe the changes in the absorption spectrum when a density is added before the cell.

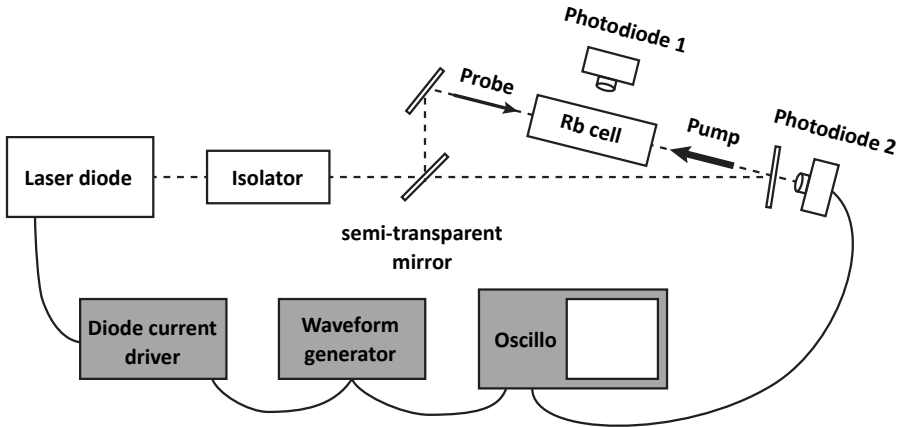
↪ Now observe what happens if you place the density after the cell, and compare it with the previous situation (density before the cell).

↪ Using the power meter provided, **measure the values of the saturation parameter**  $s = I/I_{\text{sat}}$  for the intensity incident on the cell (with or without density).

**Q5** Explain why the phenomenon of absorption saturation is clearly demonstrated qualitatively.

## 2.3 Sub-Doppler spectroscopy of hyperfine levels

The pump-probe setup is shown in Figure 4.5. The pump and probe beams must be superimposed at the gas cell.



**Figure 4.5:** Setup in pump-probe configuration..

~ Do not touch the power supply or GBF settings!

~ First place and adjust all the components, excluding the cell and photodiodes. Carefully superimpose the pump and probe beams, observing the laser spots on separating plates with the infrared viewer.

~ Using the power meter, measure the values of the saturation parameter  $s = I/I_{\text{sat}}$  for the pump and probe in this configuration.

~ Then position the cell and the photodiodes to visualise absorption on the probe beam and the fluorescence.

~ Save the displayed signals with an USB stick (or take a photo of the screen, after you've turned on "stop").

**Q6** For the two isotopes, identify the lines of the hyperfine structure (using figure 4.1). Measure the relative positions of the hyperfine lines and compare the values obtained with the expected values. Also measure the width of the observed lines.

**Q7** What do they correspond to?

In Doppler profiles covering two atomic transitions (as for rubidium 85), instead of 2 absorption lines you should observe 3 lines. The third line appears precisely in the middle of the 2 expected lines. This phenomenon is called crossover.

**Q8** Explain the origin of the crossover line. To do this, repeat the reasoning given in section 1.4 for the case of an atom with two excited states.

**Q9** Compare the absorption and fluorescence spectra. Is it possible to see the same saturation signature in the fluorescence spectra? Explain why.



## P 5

# Spectroscopy of an atomic beam

This lab illustrates the first stage of laser atom cooling experiments, namely the production of an atomic beam from an oven, which will then be captured in a magneto-optical trap. You will carry out spectroscopy of the atomic beam using a laser, most specifically the absorption and fluorescence spectroscopy of a line of the hyperfine structure of rubidium 85 around 780 nm.

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### P11 Fluorescence rate of a two-level atom.

In this tutorial, atoms will be modelled as two-level systems. The interaction between a laser beam and a two-level atom is described by the optical Bloch equations (OBE). The stationary solution of the OBEs for the fraction of atoms in the excited state is given by

$$\Pi_e = \frac{1}{2} \frac{s}{1 + s}, \quad (5.1)$$

where the saturation parameter writes

$$s = \frac{I/I_s}{1 + 4\Delta^2/\Gamma^2}, \quad (5.2)$$

as a function of the laser detuning to the atomic transition

$$\Delta = (\omega - \omega_0) - \mathbf{k} \cdot \mathbf{v} . \quad (5.3)$$

The fraction of atoms in the ground state is simply deduced from that in the excited state:

$$\Pi_g = 1 - \Pi_e . \quad (5.4)$$

In the equations above, we have used the following notations:

- $\omega$ ,  $I$  and  $\mathbf{k}$  are respectively the pulsation of the laser beam, its intensity and its wave vector;
- $\omega_0$ ,  $I_s$  and  $1/\Gamma$  are respectively the Bohr pulsation of the atomic transition, its saturation intensity and the lifetime of the excited state.
- $\mathbf{v}$  is the velocity of the atom in the laboratory reference frame.

Finally, remember that the fluorescence rate of a two-level atom is equal to  $\Pi_e \Gamma$ .

**Q10** Name the function representing the fluorescence spectrum of an atom, i.e. the change in fluorescence rate as a function of detuning. Determine its total width at half-maximum in the low-intensity limit.  $I \ll I_s$ .

**Q11** Explain why the fluorescence and absorption spectra of the laser by an atom have an identical profile as a function of the laser detuning.

**Q12** What effect causes the term  $-\mathbf{k} \cdot \mathbf{v}$  in the expression of the detuning  $\Delta$  (Equation 5.3) ?

**Q13** In a gas at equilibrium at a temperature  $T$ , the fluorescence spectrum broadened by the Doppler effect has a total width at half-maximum equal to

$$\Delta\omega = \sqrt{8\ln 2} \sqrt{\frac{k_B T}{m}} \frac{\omega_0}{c} . \quad (5.5)$$

In this equation,  $k_B = 1.38 \times 10^{-23} \text{ J/K}$  is Boltzmann's constant,  $m = 1.41 \times 10^{-25} \text{ kg}$  is the mass of a rubidium atom and  $c$  is the speed of light in vacuum. Calculate the Doppler shift expected in a cell at room temperature ( $T \sim 20^\circ\text{C}$ ) for the rubidium line centred at 780 nm.

**P12 Distribution of velocities in the atomic beam.**

The atomic beam is obtained from a rubidium vapour produced by heating a metal sample in an oven to a temperature of  $T \simeq 100^\circ\text{C}$ . This vapour escapes from the oven through a first circular orifice of diameter  $D_1 = 5\text{ mm}$ . A second orifice of diameter  $D_2 = 1\text{ cm}$  placed at a distance  $L \simeq 20\text{ cm}$  from the first one filters the distribution of transverse velocities, thus creating an atomic jet.

**Q14** Assuming that the gas is at thermodynamic equilibrium in the oven, give the energy distribution of the atoms, then their velocity distribution, expressed as a function of the norm of the velocity vector  $v = \|\mathbf{v}\|$ .

**Q15** Estimate numerically the average  $\langle v \rangle = \sqrt{9\pi k_B T / 8m}$  of the norm of the velocity vector at the exit of the oven.

**Q16** Assuming that the velocity of an atom along the axis of the circular orifices is equal to  $\langle v \rangle$ , estimate the maximum transverse velocity that will allow this atom to reach the last chamber. To make the calculation easier, consider that  $D_1 = D_2 = 1\text{ cm}$ .

## 1 Absorption spectrum

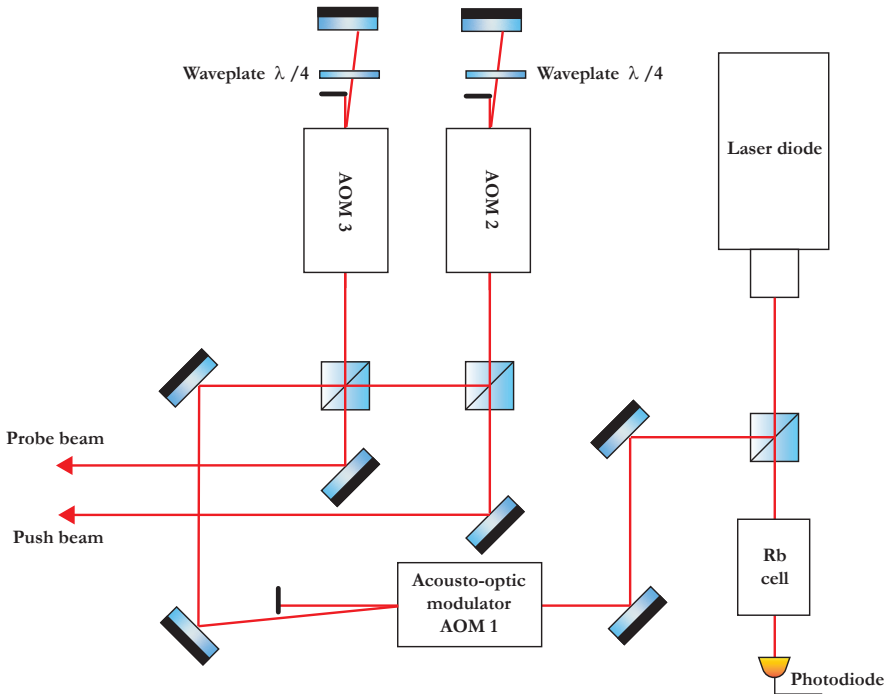
In this section you will first measure the absorption spectrum of a laser beam of wavelength  $780\text{ nm}$  through a spectroscopy cell at room temperature, then through the atomic beam produced by the oven. The optical set-up is shown in figure 5.1 below.

The natural abundance of the two isotopes of Rubidium is  $\sim 72.2\%$  for the 85 isotope and  $\sim 27.8\%$  for the 87 isotope..

**IMPORTANT** Make sure the laser diode is set to the following values: selected current (« set »)  $I_{\text{set}} = 143,5\text{ mA}$  and set temperature  $T_{\text{set}} = 19,550^\circ\text{C}$ .

### Absorption spectrum through the cell at room temperature

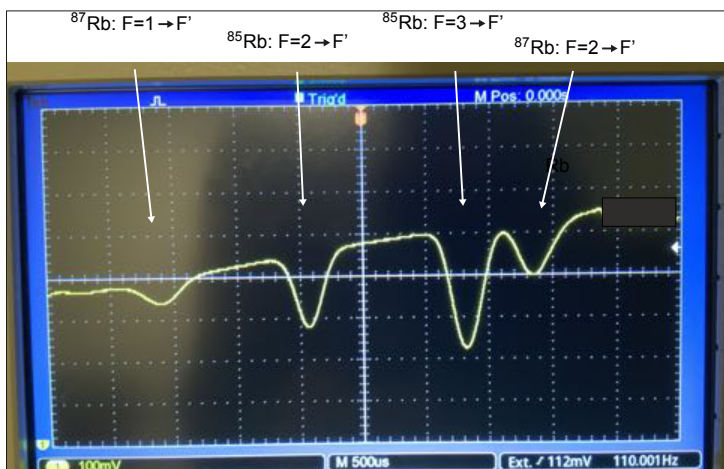
↪ Scan the laser diode current using the RIGOL function generator connected to the external modulation input of the laser diode controller and display on the oscilloscope 4 resonances such as those shown in figure 5.2. To do this, choose a symmetrical triangular sweep pattern, a modulation frequency of the order of  $100\text{ Hz}$  and a peak-to-peak modulation amplitude of the order of  $300\text{ mV}$  (and a zero offset voltage).



**Figure 5.1: Description of the optical table.** The light source is a laser diode emitting around 780 nm. Part of the beam is used for spectroscopy in the glass cell. The other part of the beam is sent through a first single-pass AOM. The laser beam is then split into two beams, each injecting a double-pass AOM. The probe and push beams thus generated are sent to the vacuum chamber.

Each of these resonances actually comprises several unresolved atomic transitions: the difference in energy between the hyperfine sub-states of the excited state is smaller than the Doppler shift at ambient temperature. An energy diagram of the hyperfine structure of rubidium isotopes 85 and 87 is given in the appendix.

**Q17** The lines observed in Figure 3 are identified through the isotope (85 or 87) and the  $F$  hyperfine state of the ground state. Justify this identification by checking that the relative distance between the two lines of each isotope



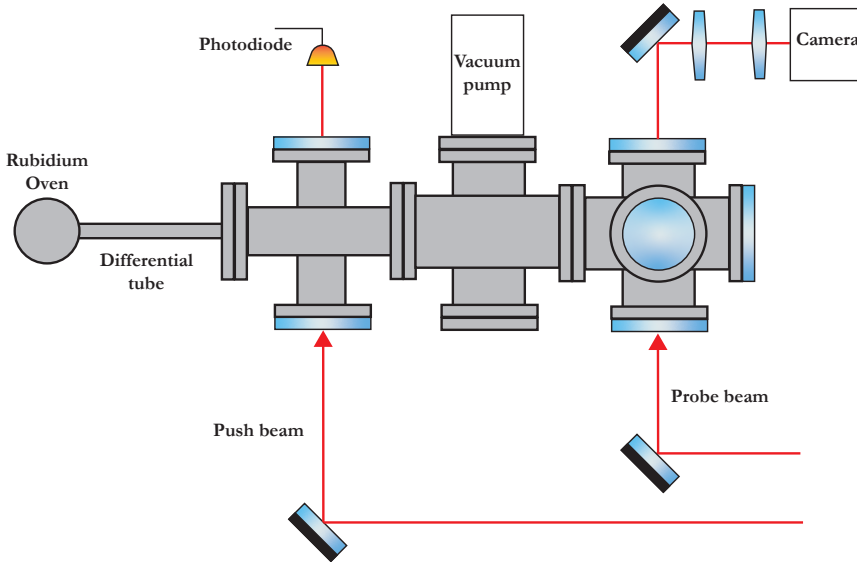
**Figure 5.2: Absorption spectrum in the cell.** This spectrum was obtained using the same set-up as the one you have, by measuring the laser intensity transmitted through the cell using an amplified photodiode and sweeping the frequency of the laser diode. It shows 4 lines linked to the hyperfine structure of the 85 and 87 isotopes of rubidium around 780 nm (see the energy diagrams in the appendix).

corresponds to what is expected from the energy diagrams.

**Q18** Using the same data, determine the coefficient of proportionality between the current modulation voltage and the optical frequency of the laser.

**Q19** Measure the total width at half-maximum of the two rubidium 85 resonances. Compare your result with the natural width expected for a transition, i.e.  $\Gamma/2\pi = 6,1 \text{ MHz}$ , and with the width calculated in question Q4. Comment your result.

**Absorption spectrum through the atomic beam** In this section, you will measure the absorption spectrum of a jet of rubidium atoms using the probe beam shown in figure 5.3.



**Figure 5.3: Description of the vacuum chamber.** The jet of rubidium atoms is produced in an oven heated to around  $100^\circ\text{C}$  and whose outlet is bounded by a narrow cylinder. The jet propagates from left to right in the diagram. The transverse velocity distribution of the jet is characterised in the rightmost chamber, either by absorption of the probe beam using a photodiode (not shown), or by fluorescence induced by the same probe beam using a camera. In the intermediate chamber, a push beam can deflect the atomic beam before it reaches the probe.

**Initial settings.** The AOM2 and AOM3 acousto-optic modulators are driven using the SIGLENT arbitrary function generator, which has two output channels. It will be ensured for both output channels that a sinusoid of frequency 200 MHz and amplitude 100 mV is sent to the two acousto-optic modulators.

~ Simultaneously observe the signals from the photodiodes in the cell and the jet using an oscilloscope.

**Q20** What obvious difference(s) can you see between the two absorption signals?

~ Modify the amplitude of the laser diode current sweep so that only the most intense absorption line is observed in the cell. The absorption spectrum from the atomic beam should now look like that shown in figure. 5.4.



**Figure 5.4: Absorption spectrum through the atomic jet.** This spectrum was obtained using the same set-up as the one you have, by measuring the laser intensity transmitted through the vacuum chamber and the atomic jet, using an amplified photodiode and sweeping the frequency of the laser diode.

**Q21** Using the energy diagrams given in the appendix, identify the atomic transitions visible in the absorption spectrum of the atomic beam.

**Q22** Measure the total width at half-maximum of the most intense absorption line across the jet.

From the expression for Doppler broadening given in equation 5.5, estimate an "effective" temperature corresponding to your linewidth measurement. Why can't this be a 'true' temperature?

**Q23** Discuss the value of the atom jet for spectroscopic measurements.

**Q24** Give a rough estimate of the frequency resolution achieved by external modulation of the laser diode current.

In the next section, we will see that this resolution limit can be greatly reduced by using an acousto-optic modulator (AOM) to sweep the laser beam frequency.

## 2 Fluorescence spectrum

You are now going to observe the fluorescence spectrum of the atomic jet using a camera positioned perpendicular to the probe beam.

~ Using modulation of the laser diode current to vary the frequency of the laser beam (procedure identical to the previous section), observe the fluorescence resonance on the camera.

**Q25** Determine the orientation of the probe beam and the atomic beam on the camera image.

~ Disable the laser diode current sweep and add a  $50\ \Omega$  termination to the controller's external modulation input. Slowly modify (a few MHz at a time) the frequency of AOM No. 3 driven by the SIGLENT generator until the fluorescence resonance on the camera is restored.

~ Determine the frequency range that needs to be covered to visualise the entire fluorescence resonance, i.e. the minimum (respectively maximum) frequency below (respectively above) which the fluorescence signal is negligible.

**Q26** What is the first observation you can make about the resolution of the frequency sweep performed using the AOM compared with that performed by modulating the laser diode current?

~ We want to measure the shape of the observed resonance precisely. To begin, acquire a fluorescence image using the camera control software. Elect an area of interest (ROI) covering the entire sensor in the direction of the laser beam but only the fluorescence signal in the direction of the jet. Extract the average number of grey levels  $N_{n.g.}$  calculated by the histogram function.

~ We also want to ensure that we are working within a linear response regime, defined by a saturation parameter  $s$  less than the unit. To determine this regime in the experiment, we begin by choosing the AOM frequency for which the fluorescence signal (i.e. the average number of grey

levels  $N_{\text{n.g.}}$  in the ROI) is maximum. Keeping this frequency fixed, we vary the amplitude of the signal sent to the AOM by the SIGLENT generator: for each value of the amplitude chosen, we measure both  $N_{\text{n.g.}}$  and the voltage  $V_{\text{PD}}$  on the photodiode of the probe beam.

**Q27** Plot the curve  $N_{\text{n.g.}}$  as a function of  $V_{\text{PD}}$  and identify the linear and saturated fluorescence regimes. Determine the maximum value of amplitude of the AOM drive signal below which the variation of  $N_{\text{n.g.}}$  is linear with  $V_{\text{PD}}$ .

~ Measure the number of grey levels in the ROI as a function of the frequency of the AOM drive signal. Choose about ten frequency values spread over the range you determined in question Q16.

**Q28** Plot the fluorescence spectrum obtained as a function of the AOM drive frequency. Determine its total width at half-height and compare it with that of the absorption spectrum by the atomic beam. Be careful to take into account the double passage through the AOM, which shifts the laser beam frequency by twice the AOM drive frequency!

~ When the drive frequency of the AOM is varied, the diffraction efficiency changes as we move away from the Bragg condition. This effect is likely to alter the measurement of the fluorescence spectrum and can be corrected. To do this, measure  $V_{\text{PD}}$  as a function of AOM frequency over the range used for the previous measurement in order to calibrate the variation in optical power of the probe beam. Correct the fluorescence spectrum measurement to compensate for the effect of the variation in probe beam power.

**Q29** Plot the corrected spectrum and compare it with the raw spectrum. Does it seem necessary to take into account the modulation of the optical power of the probe beam?

~ Reproduce the measurement of the fluorescence spectrum for the maximum value of the optical power of the probe beam and plot the new spectrum obtained.

**Q30** Compare the width of this spectrum with that of the spectrum obtained at low power and comment on the effect of saturation.

### 3 OPTIONAL SECTION - Mechanical action of light on atoms

*If you have time, you can deal with this optional section.*

In this final section, we study the mechanical effect of laser light on the atomic jet. This effect is similar to that exerted by sunlight on the tail of a comet: the radiation pressure exerted by the sunlight bends the tail of the comet.

The radiation pressure force exerted by the laser light on the atom at two levels is written,

$$\mathbf{F}_{\text{rad}} = \hbar \mathbf{k} \frac{\Gamma}{2} \frac{s}{1+s} . \quad (5.6)$$

↪ Add the pusher beam to the empty chamber at the end of the oven. Set the power of the push beam to its maximum and vary its frequency until an effect on the fluorescence resonance is observed.

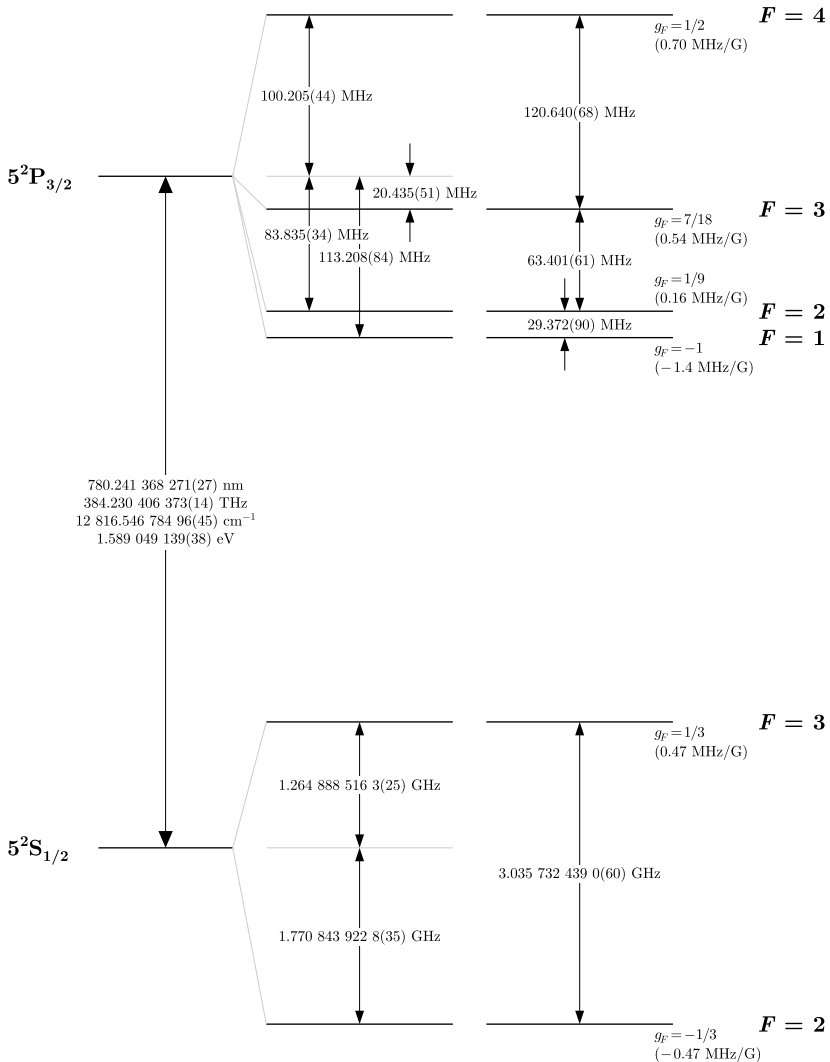
**Q31** Interpret the modification of the spectrum by the mechanical action of the laser beam at the oven outlet.

**Q32** Estimate the variation in impulse experienced by an atom passing through a laser beam.

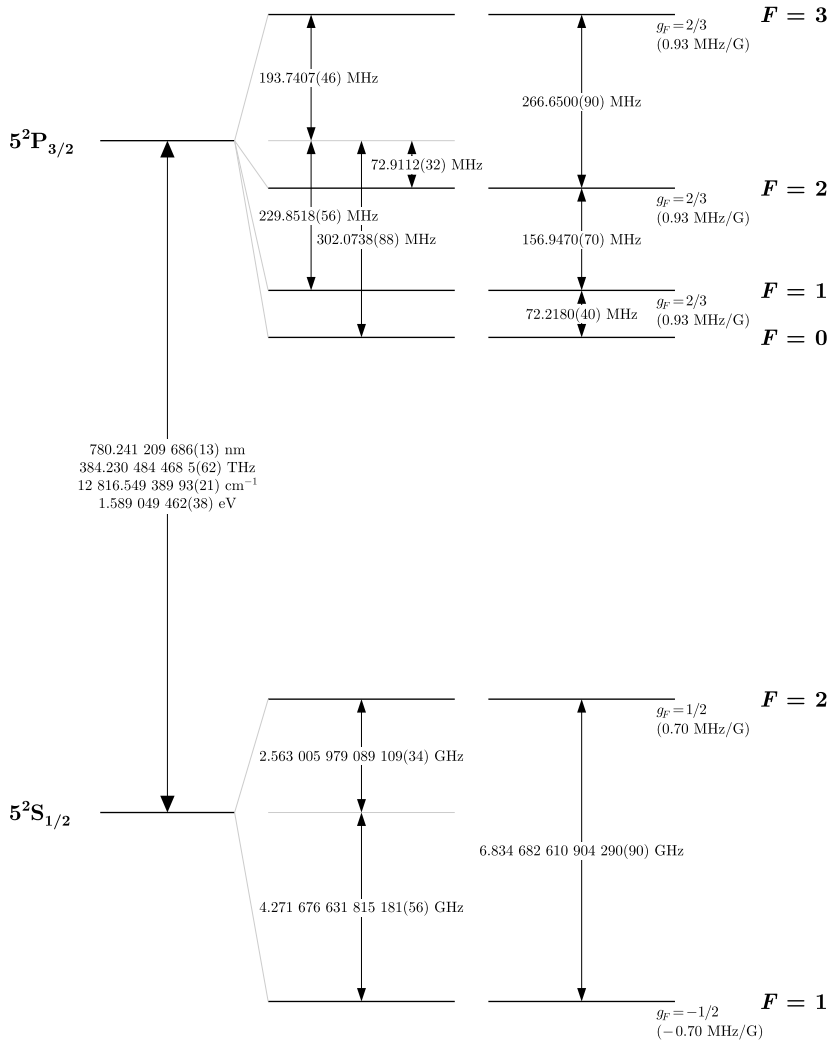
**Q33** Discuss the best beam size for measuring the velocity distribution of the atomic beam.

## 4 Annexe

Hyperfine structure of the D2 line of rubidium isotopes 85 and 87. The figures are taken from documents posted online by Daniel Steck at the following address <http://steck.us/alkalidata/>.



**Figure 5.5:** Hyperfine structure of the D2 transition of rubidium 85.



**Figure 5.6:** Hyperfine structure of the D2 transition of rubidium 87.

## P 6

# Single-Photon Interferences : Grangier-Roger-Aspect experiment

## Contents

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## 1 Introduction

The Grangier-Roger-Aspect (GRA) experiment provided a perfect illustration of wave-corpuscle duality, namely the ability of individual quantum systems, referred to as particles - in this case, a single photon - to form interference, i.e. an experimental signature typically associated with waves.

This experiment was carried out in 1985-1986. The notion of single-photon sources, and how to approach them experimentally, was a relatively emerging subject. In particular, the question was whether a sufficiently attenuated light source could constitute a single-photon source, in the quantum sense of the term (the answer is no: see the Appendix.).

The Grangier-Roger-Aspect (GRA) experiment was carried out at Orsay, in the former 503 building of Institut d'Optique, following on from the Bell inequality violation experiments. In particular, it exploited the same photon source, which was in fact a source of *pairs* of photons. The designers of the GRA experiment were the first to produce a *heralded photon source*. Each of

the two beams emanating from the twin-photon source does not initially have the character of a single-photon source. However, based on measurements of the first beam (containing "heralding" photons), the second beam can be conditioned to exhibit single-photon flux behavior.

The aim of this labwork session is therefore both to study the operating principle of such a source and to use it to reproduce the results of the GRA experiment.

## 1.1 The corpuscular character of the photon

### What is a single photon?

Let's consider the  $\mathcal{E}$  electromagnetic field occupying a certain volume  $V$  of space. Solving Maxwell's equations within this volume  $V$ , combined with boundary conditions, allows us to decompose this field  $\mathcal{E}$  as a superposition of  $\mathcal{E}_m$  modes.

In classical physics, the energy of each of these field modes can take on arbitrary values, related to the square modulus of their amplitude  $|\mathcal{E}_{\uparrow}|^2$ . In quantum physics, field modes cannot take on arbitrary energy values: the energy of electromagnetic field modes is quantized, according to the same rules as quantum harmonic oscillators. The energy  $E_m$  of the  $m$  mode of frequency  $\omega$  writes :

$$E_m = \hbar\omega\left(n + \frac{1}{2}\right)$$

where  $n$  is the number of mode excitations, also called *quanta*. It is convenient to use the concept of Fock states, or number states, to designate the quantum state of a field. A field containing exactly  $n$  photons is then denoted  $|n\rangle$ .

**A photon is a single excitation, a quantum of a mode of an electromagnetic field .**

In general, a pure quantum field state can be written as a superposition of Fock states:

$$|\Psi\rangle = \sum_i c_i |i\rangle$$

i.e. as a superposition of 0-photon states (vacuum), 1-photon states, 2-photon states, etc...

A single photon refers to the quantum state of a mode of the electromagnetic field in its first excited state: it is more formally written as  $|1\rangle$  – and by extension, the word photon also refers to the  $\hbar\omega$  amount of energy associated with this excitation.

An ideal single-photon source is therefore a light source producing a field whose state, when measured, is systematically found to be in the  $|1\rangle$  state. The quantum state of the field produced by this source is therefore a pure state written :

$$|\Psi_{\text{SPS}}\rangle = |1\rangle$$

where SPS stands for *Single Photon Source*).

Conversely, the other quantum states of light can be measured in multiphoton states  $|n\rangle$  with  $n \geq 2$ : for a pure state, Fock's decomposition into states includes terms in  $|2\rangle$ ,  $|3\rangle$ , and so on. **Most importantly, this property is true even for extremely attenuated sources.**

A brief discussion of other types of quantum states commonly used to describe different light sources, which are not single-photon sources, such as thermal sources and coherent sources, is given in the Appendix.

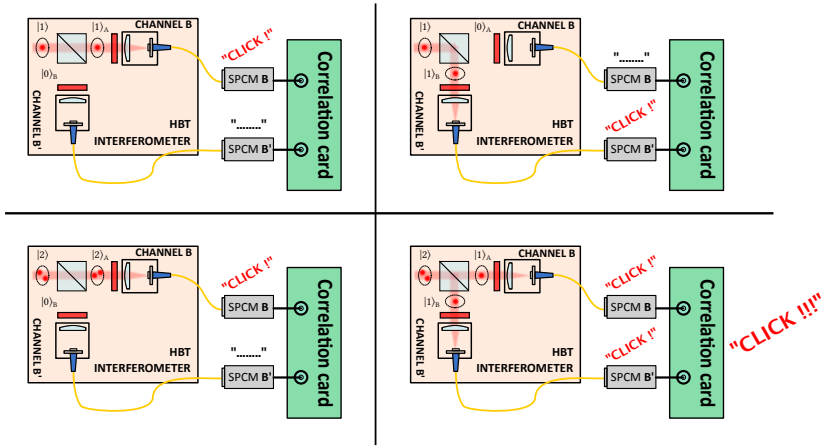
### Principle of HBT interferometry

The experimental demonstration of the corpuscular nature of a single-photon source is based on a principle: **a single photon can only be detected at a single location.** A way to proceed is therefore by measuring coincidences between two detectors placed at two different locations.

The adequate setup is that of a **Hanbury-Brown and Twiss interferometer, usually abbreviated to HBT** (see Figure 6.1). It consists of a simple 50/50 splitter, whose two output ports lead to two avalanche photodiodes, denoted  $A$  and  $B$ , placed at equal distance from the splitter.

A coincidence measurement consists in measuring the probability that the two channels  $A$  and  $B$  detect a photon (almost) simultaneously. Two cases are of interest here:

- For an ideal single-photon source, the electromagnetic field of the mode contains only a single photon at each instant. The  $|1\rangle$  photon can be transmitted or reflected by the splitter - but never split into two components on each of the  $A$  and  $B$  paths. When the detectors are well synchronized, it is therefore impossible to detect coincidences with an ideal single-photon source. This is known as **photon antibunching**.
- If the light source has multi-photon state components, such as  $|2\rangle$ , each photon can be reflected or transmitted as it passes the splitter. So, 50% of the time, one photon is detected on channel  $A$  and the other on channel  $B$  : **any light source other than a single-photon source gives rise, at least occasionally, to coincidences.**



**Figure 6.1:** Principle of the experimental test of the statistical nature of a source by coincidence measurements. A single-photon source at the input to an HBT interferometer never generates coincidences, unlike other sources which contain multiphoton states

A more detailed discussion of the formalism of the quantum theory of photodetection applied to the case of coincidence detection is given in the Appendix.

### Parameter $g^{(2)}(0)$

An HBT interferometer measures an objective parameter to characterize the statistical properties of the source: the intensity autocorrelation function  $g^{(2)}(0)$ . In the classical formalism, this function evaluates the intensity fluctuations of a light source.

$$g^{(2)}(\tau) = \frac{\langle \hat{n}(t) \hat{n}(t + \tau) \rangle}{\langle \hat{n} \rangle^2}$$

The interferometer's separator, assumed to be lossless and perfectly balanced, has two output channels enabling us to evaluate the  $g^{(2)}$  function of the initial mode by simultaneous measurements on the two distinct output modes  $A$  and  $B$ . Using the number operators attached to the two channels, we write :

$$g^{(2)}(0) \equiv g_{BB'}^{(2)}(0) = \frac{\langle \hat{n}_B \hat{n}_{B'} \rangle}{\langle \hat{n}_B \rangle \langle \hat{n}_{B'} \rangle}$$

At zero delay, the function  $g^{(2)}$  is interpreted as the ratio of the probability of observing coincidences  $P_{AB}$ , to the individual probabilities of ob-

serving detection events,  $P_B$  and  $P_{B'}$ . These probabilities are directly proportional to the count rates. We denote with the letter  $R$  the count rates in [counts.s<sup>-1</sup>] and with the letter  $N$  the number of counts during the integration time  $\Delta t$  in [blows], we have  $N_B = R_B \Delta t$  and  $N_{B'} = R_{B'} \Delta t$  and coincidences  $N_{BB'} = R_{BB'} \Delta t$  between  $B$  and  $B'$ .

$$g_{AB}^{(2)}(0) = \frac{P_{BB'}}{P_B P_{B'}} = \frac{N_{BB'}}{N_B N_{B'}} = \frac{R_{BB'}}{R_B R_{B'} \Delta t}$$

Since the number of coincidences for a single-photon source is zero, the expected theoretical result is  $g_{BB'}^{(2)}(0) = 0$  – and more generally, a value of  $g_{BB'}^{(2)}(0)$  significantly smaller than 1 is proof of the quantum, anti-bunched nature of photons. Conversely, any combination of photon fluxes from thermal or coherent sources can only yield  $g_{BB'}^{(2)}(0) \geq 1$ .

## 1.2 Single-photon interference

The evolution of quantum states, even those associated with objects described as “corpuscular”, is described by the Schrödinger equation, which is intrinsically a *wave* equation. It is therefore possible to observe interference phenomena - which is the hallmark of undulatory objects - even with a flux of single photons.

To observe interference, it's convenient to use...an interferometer. For the sake of simplicity, we discuss here the case of a single-photon source, providing  $|1\rangle$  states sent to the input of a Mach-Zehnder interferometer (see Figure 6.2)<sup>1</sup>.

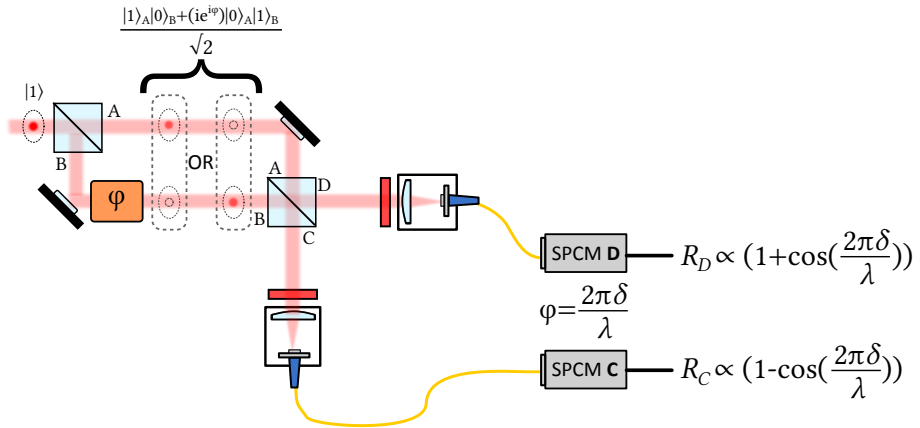
The quantum state after the first splitter is simply written:

$$|\Psi_{\text{SPS}}^{\text{in},1}\rangle = |1\rangle \longrightarrow |\Psi_{\text{SPS}}^{\text{out},1}\rangle = \frac{|1_A\rangle|0_B\rangle + i|0_A\rangle|1_B\rangle}{\sqrt{2}}$$

where  $i$  appears as the phase factor between reflection and transmission coefficients, ensuring the unitarity of the transformation. It is possible to adjust the length of the interferometer arms, so as to introduce a step difference  $\delta$  between the two arms, associated with a phase shift  $\varphi = \frac{2\pi\delta}{\lambda}$ . The input state of the separator closing the interferometer is written :

---

<sup>1</sup>The substance of the discussion is the same for Michelson, Fizeau or Young's slit interferometers!



**Figure 6.2:** Mach Zehnder interferometer illuminated by a single photon state at the input of the first splitter. The quantum state of the field is written as a superposition of states once inside the interferometer. The two outputs of the Mach-Zehnder are connected to avalanche photodiodes: the average hit rates on each channel display the same dependence on the step difference as the intensity observed using a classical input light source: they are the same two-wave interference fringes

$$|\Psi_{\text{SPS}}^{\text{in},2}\rangle = \frac{|1_A\rangle|0_B\rangle + ie^{i\varphi}|0_A\rangle|1_B\rangle}{\sqrt{2}} \quad (6.1)$$

$$\longrightarrow |\Psi_{\text{SPS}}^{\text{out},2}\rangle = \frac{1}{\sqrt{2}} \left[ \left( \frac{|1_C\rangle|0_D\rangle + i|0_C\rangle|1_D\rangle}{\sqrt{2}} \right) + ie^{i\varphi} \left( \frac{i|1_C\rangle|0_D\rangle + |0_C\rangle|1_D\rangle}{\sqrt{2}} \right) \right] \quad (6.2)$$

$$= \left( \frac{1 - e^{i\varphi}}{2} \right) |1_C\rangle|0_D\rangle + \left( \frac{1 + e^{i\varphi}}{2} \right) |0_C\rangle|1_D\rangle \quad (6.3)$$

As in classical physics, the field at the output of an interferometer is written as a recombination of different amplitudes that have passed through the two arms of the interferometer. The number of counts measured by a detector per unit of time is then directly proportional to the probability of detection on the same channel. On channel  $C$ , for example :

$$R_D \propto P_D = |\langle 0_C | \langle 1_D | \Psi_{\text{SPS}}^{\text{out},2} \rangle|^2 = \frac{1}{2} (1 + \cos \frac{2\pi\delta}{\lambda})$$

The count rate is written as the result of two-wave interference, and therefore evolves sinusoidally with the optical path difference between the two arms. A single photon is therefore...a wave!

### 1.3 Heralded photon source

The set-up used is based on a spontaneous parametric conversion process in a BBO crystal. Illuminated by a pump beam from a laser diode at 405 nm, the crystal generates two beams containing twin photon pairs at 810 nm, a signal beam ( $s$ ) and an idler beam ( $i$ ).

The pump is in a coherent state, comprising different number states. By converting pump photons, the crystal can generate photon pairs, but it can also generate double pairs, triple pairs, etc. from several pump photons. Formally, the quantum state in each beam is a correlated mixture of number states between the two arms. For the sake of simplicity, we write it here as a linear combination of several Fock states<sup>2</sup>:

$$|\Psi_{SPDC}\rangle = \sum_{i=0}^{+\infty} c_n |n\rangle_s |n\rangle_i = c_0 |0\rangle_s |0\rangle_i + c_1 |1\rangle_s |1\rangle_i + c_2 |2\rangle_s |2\rangle_i + \dots$$

On each of the  $s$  or  $i$  channels considered independently, we do *not* have a single-photon flux, but a superposition with multi-photon components: the expected value of  $g_{AB}^{(2)}(0)$  is greater than 1.

---

<sup>2</sup>Refer to the Appendix for an advanced discussion of the quantum state generated by the crystal

The correlation properties between the two beams in state  $|\Psi_{SPDC}\rangle$  are very interesting: if, when taking a measurement on channel  $i$ , we find the field in a  $|1\rangle_i$  state, then, by projection and necessarily, the corresponding field on channel  $s$  at the exact instant of this measurement must be in the  $|1\rangle_s$  state...:

$$\langle 1|_i \Psi_{SPDC}\rangle = |1\rangle_s$$

We speak of **conditioning the measurement**.

Experimentally, conditioning is carried out by recording coincidence events. The detection of a photon on the  $i$  channel generates a short coincidence time window in an electronic circuit. This is known as a **heralding photon**: the window represents the brief interval of time during which the quantum state of the  $s$ -channel field is perfectly known, and is precisely a single photon  $|1\rangle_s$ . This photon is the heralded photon.

Measurements are then carried out on the  $s$  channel - in this case, in this labwork session, by sending it through a Michelson interferometer and then an HBT interferometer. After having equalized the optical and electronic delays between the heralding channel  $i$  and the signal channel  $s$ , we select only those events that give rise to coincidences between the two channels. These events correspond to measurement results for the  $s$  channel performed only on a  $|1\rangle_s$  state. It is as if we had used a single photon source in our set-up: we call it a “heralded photon source”.

## 2 Experiments

### 2.1 Presentation of the setup

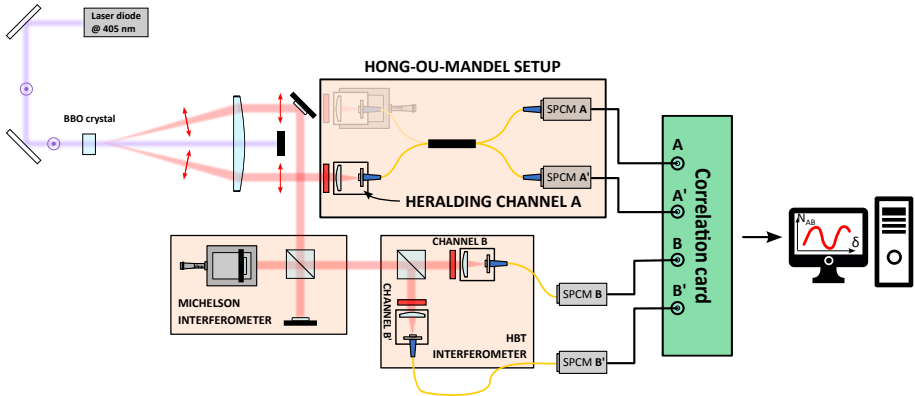


Figure 6.3: Optical assembly diagram.

The experimental set-up is an extension of the setup dedicated to the study of the HOM effect.

A laser diode at 405 nm pumps a BBO crystal, generating two beams along the edges of a cone whose apex angle is close to 3 degrees. One beam is sent to a fibered collimator, whose output is connected to an avalanche photodiode. This is the heralding channel, denoted *A*.

Photons from the second, "heralded" channel are reflected by a mirror to a Michelson interferometer, with one mirror mounted on a piezoelectric translation stage.

The Michelson's output channel is connected to an HBT interferometer, whose output channels *B* and *B'* feature fibered collimators connected to avalanche photodiodes.

Channels *A*, *B*, and *B'* are connected to an ALTERA DE2 FPGA board, which records the detection rates per time unit on each channel, and calculates the different coincidence rates of interest:

The FPGA board can process signals from up to 4 different detectors: we're using three here. The board sends back to the computer an array of up to 8 32-bit numbers, representing the data from 8 different counters. The first 4 elements of the array are the single counts from up to 4 detectors (counter 0 - counter 3), in this order: *A*, *B*, *A'*(not used), *B'*. The last 4 elements are the

|      | SW0  | SW1  | SW2  | SW3  |
|------|------|------|------|------|
| AB   | 1    | 1    | 0    | 0    |
|      | SW4  | SW5  | SW6  | SW7  |
| ABB' | 1    | 1    | 0    | 1    |
|      | SW8  | SW9  | SW10 | SW11 |
| AB'  | 1    | 0    | 0    | 1    |
|      | SW12 | SW13 | SW14 | SW15 |
| BB'  | 0    | 1    | 0    | 1    |

**Table 6.1:** Tableau résumant les positions attendues des interrupteurs pour le VI coincidence, *s232.vi*.

coincidence counts (Counter 4 - Counter 7). Coincidences are determined by the settings of switches SW0 - SW15 on the DE2 board. Each counter uses four switches to determine which coincidence it counts - counter 4 uses switches SW0 - SW3, counter 5 uses switches SW4 - SW7, etc. When a switch is activated, the corresponding LED lights up. With this arrangement, any of these 4 counters can determine any arbitrary coincidence between 2, 3 or 4 detectors. In our case, different coincidences are counted: to correctly retrieve the information, make sure switches SW0 and SW1 are on, while SW2 and SW3 must be off - switches SW4 to SW15 are only important in the case of a 4-detector configuration (see Table 6.1).

- The rates  $R_A$ ,  $R_B$ ,  $R_{B'}$  are the raw, unconditioned count rates recorded on the different detection channels.
- The rates  $R_{AB}$  and  $R_{AB'}$  are the detection rates conditioned by the heralding channel - in practice, these are the coincidences between  $A$  and  $B$ , or  $A$  and  $B'$ , recorded at the Michelson output, with the heralding process by  $A$  allowing selection of the "single photon" states.  $R_{AB}$  and  $R_{AB'}$  therefore correspond to the interference signal in equation (1.2).
- The rate  $R_{BB'}$  is the unconditioned coincidence rate between the two channels of the HBT interferometer. It can therefore be used to determine the value of  $g^{(2)}(0)$  for one of the two beams from the SPDC process, which is not a single-photon source.
- The rate  $R_{ABB'}$  is a 3-detector coincidence rate, and is interpreted as the HBT interferometer coincidence rate, between  $B$  and  $B'$ , conditioned by the heralding channel  $A$ . It allows us to trace back to the value of  $g^{(2)}(0)$ , when the beam state is placed in the single-photon state. It is this very rate that demonstrates the source's antibunching properties.

In a three-detector configuration, the experimental definition of  $g^{(2)}$  must be modified. Since the relevant detection events can only occur when an electronic gate is opened by the heralding channel, the count and coincidence probabilities are equal to the number of events divided by the total number of generated gates, i.e. normalized by the number of detections on the heralding channel.

$$P_B = \frac{N_{AB}}{N_A} \quad P_{B'} = \frac{N_{AB'}}{N_A} \quad P_{BB'} = \frac{N_{ABB'}}{N_A}$$

so that we obtain :

$$g_{ABB'}^{(2)}(0) = \frac{N_{ABB'} N_A}{N_{AB} N_{AB'}}$$

For an ideal photon source,  $N_{AB} \neq 0$  because a SPDC beam is not a stream of single photons, but we have  $N_{ABB'} = 0$ , and therefore  $g_{3\text{det.}}^{(2)}(0) = 0$ .

## 2.2 Settings

### Alignment of the Michelson interferometer

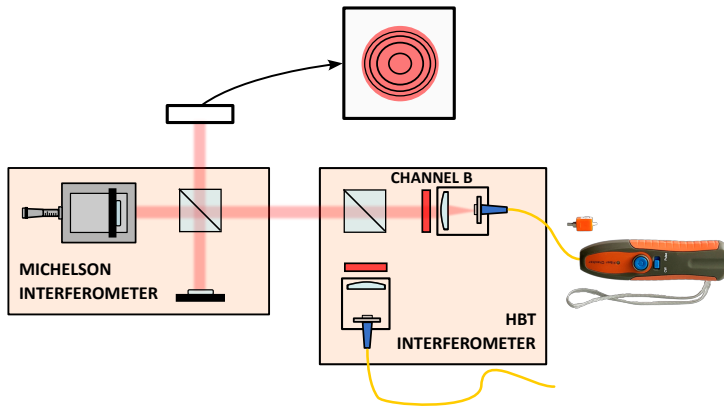
**This session is an extension of the TP HOM session. The alignment procedure described below assumes that this part of the set-up is correctly adjusted and allows observation of the HOM “dip”, the signature of two-photon interference between the two SPDC beams.**

↪ Switch on the piezoelectric stage, then run Thorlabs’ Kinesis software to control its position.

↪ **WITH UTMOST CARE, AND WITHOUT TOUCHING ITS SURFACE**, flip the mirror mounted on...its *flip mount* to reflect the photons present in the left part of the emission cone.

Before you start acquiring measurements, it is crucial to check that the Michelson interferometer is correctly aligned and that the photons leaving the *BBO* crystal are sent to their respective detectors. The precision of the adjustment must be extremely fine to be certain that the photon sent to the Michelson is indeed the twin of the photon sent to the heralding channel *A*.

The first step is to check the Michelson’s alignment, by making sure that interference fringes are indeed observed on the Michelson’s output channel. To do this, we use a fiber checker, connected to the output of an optical fiber linked to the collimator. This fiber checker contains a laser diode emitting at around  $750\text{ nm}$ , and enables a beam to be sent back from the collimators, passing through the Michelson interferometer and the rest of the setup in the



**Figure 6.4:** Use the fiber checker to align the Michelson in the opposite direction

opposite direction. This beam is intense and visible, so it will be possible to observe fringes at the output of the interferometer if the setting is close to optical contact).

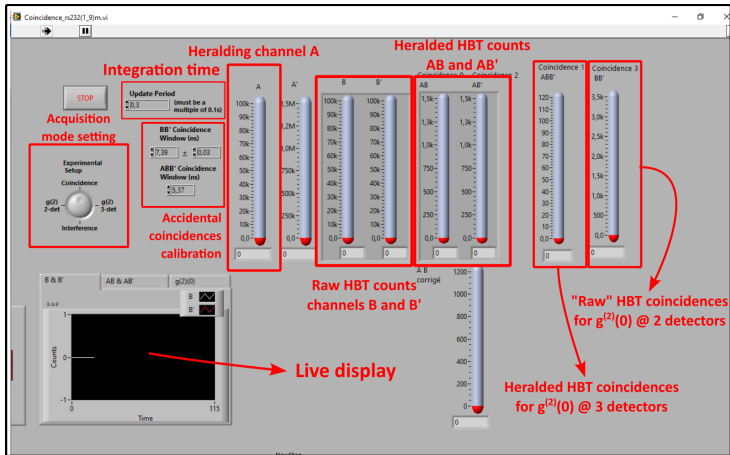
**CAUTION !** Fibers need to be handled with the utmost care, and as infrequently as possible. For this reason, NEVER disconnect the fiber outputs connected to the collimators, as this operation is never necessary.

↪ Check that the power supply to the photodiodes is switched off. Then, on the photodiode housing, disconnect the fiber corresponding to the *B* detection channel.

↪ Connect the checker to channel *B*. Don't forget to replace the plug on the corresponding photodiode input so as not to leave it exposed.

↪ Place a converging lens at the Michelson's input: it forms a cone of incidence angle at the interferometer's input. Then place a screen at the output of the interferometer. If the set-up is not too out of alignment, you should obtain interference rings. If necessary, adjust the M1 mirror (the fixed one, not mounted on the wedge) to superimpose the beams as best you can, then obtain rings by achieving parallelism between the mirrors.

↪ From a uniform air gap configuration, scroll through the rings using the piezoelectric translation stage to get as close to the optical contact as experimentally possible, trying to achieve a uniform, flat interference pattern.



**Figure 6.5:** Fenêtre du VI Labview de mesure des coïncidences.

↪ Once this has been done, check that connecting the checker to channel  $B'$  produces the same interference pattern. If necessary, gently adjust the of the collimator to obtain fringes similar to the previous situation.

### Fine collimator alignment

Now that the Michelson is properly aligned, we need to check that the two collimators at the output of the HBT assembly are both able to intercept the same photons from the BBO crystal.

**P1** Explain why we need to maximize the number of coincidences detected and not necessarily the number of single hits received on the detectors.

↪ Remove the checker, and reconnect all the fibers to the detectors.

↪ Switch on the pump diode temperature control box and start the servo drive.

↪ **CAUTION** : Always switch on the coincidence circuit power supply **BEFORE** switching on the avalanche photodiodes, and switch off the photodiodes before switching on the stabilized power supply!

turn on the coincidence circuit power supply of the ALTERA DE2 board.

On the computer, launch the Labview VI “Coincidences.vi.” and start acquisition.

↪ With the room lights off, switch on the power supply to the photodiode box and observe the appearance of a signal on the interface.

↪ With the room lights off, what do the counts generated by the photodiodes correspond to?

↪ Turn on the laser diode current control box. Starting with zero current, gradually increase the current while monitoring the number of strokes detected by the detectors: this should be in the order of a few thousand to a few tens of thousands.

**CAUTION : avalanche photodiodes are EXTREMELY FRAGILE! Always ensure that the rate of detected counts is well below one million counts per second!**

↪ On the Labview interface, use the knob selector to choose the "g(2) 3 det." mode. Read off the number of counts on the different channels, as well as the number of coincidences  $AB$  and  $AB'$ .

↪ Place a screen to cut off one of the Michelson's two channels. Adjust the alignment of a first collimator, trying to maximize the number of coincidences. The settings are very sensitive, so be careful not to lose all your coincidences!

↪ Optimize the setting of the second collimator, then block the other Michelson channel. Wherever possible, adjust the collimators and slightly rewind the Michelson mirrors to maximize and balance the number of coincidences received by the two collimators from the fluxes of the two Michelson channels.

If no coincidences or counts are visible on the photodiodes, it is best to first find a suitable approximate position for the collimators without trying to find even a weak signal and losing the alignment. The checker can then be used in reverse propagation, from the HBT detection channels  $B$  and  $B'$ , and from heralding channel  $A$ . We then use the degrees of freedom of the collimators to observe that the beams from  $B$  and  $B'$  are well focused in the BBO crystal, at the same location as heralding channel  $A$ . This roughly ensures that the collimator collects a few photons from the crystal, providing a starting point for optimizing the setting.

## 2.3 Measurements

Recall that the source's intensity auto-correlation coefficient is written :

$$g_{\text{GRA}}^{(2)}(0) = \frac{N_{ABB'} N_A}{N_{AB} N_{AB'}}$$

If it is zero, for an ideal single-photon source, the expected *experimental* value can be modeled and is different from 0: to do this, we need to take into account

that, under realistic conditions (coincidence window of non-zero size, detector efficiency, etc.), several events are actually recorded as triple  $N_{ABB'}$  or double  $N_{AB}$  and  $N_{AB'}$  coincidences :

- **Accidental double coincidence** As in the HOM experiment, double coincidences arising from the detection of two independent photons on two detectors during the coincidence window must be taken into account. The number  $N_{acc}^{(1+1)}$  is written :

$$N_{acc}^{(1+1)} = (N_A N_B + N_A N_{B'}) \frac{\Delta t}{T_{int}^2}$$

So the total number of coincidences measured experimentally is  $N_{AB} = N_{AB,source}^{(2)} + N_{AB,acc}^{(1+1)}$ . By definition, a source of pairs is highly correlated and naturally produces large numbers of coincidences. When the "xperiment is correctly set up, the number of accidental coincidences can be neglected before the number of desired coincidences, those of the source pairs:  $N_{AB} \approx N_{AB,source}^{(2)}$ .

- **Purely accidental triple coincidence** : Three photons from different, and therefore independent, pairs reach the detectors on channels  $A$ ,  $B$  and  $B'$  during the same coincidence window  $\Delta t$ . The number  $N_{acc}^{(3)}$  of this accidental triple coincidence counted over the time  $T_{int}$  wrties :

$$N_{acc}^{(3)} = \frac{N_A N_B N_{B'} \Delta t^2}{T_{int}^3}$$

Where  $T_{int}$  is the integration time.

- **Normal double coincidence with an accidental third count**: In this case, two photons from a pair generate a coincidence on paths  $A$  and  $B$  or paths  $A$  and  $B'$  and a count is recorded on the third detector during the coincidence window. The number  $N_{acc}^{(2+1)}$  of accidental triple coincidences events recorded during the time  $T_{int}$  is written :

$$N_{acc}^{(2+1)} = (N_{AB} N_{B'} + N_{AB'} N_B) \frac{\Delta t}{T_{int}^2}$$

The coincidence window is chosen to be very small compared to the measurement time, on the order of a few nanoseconds. Moreover, for a source of photon pairs, we have  $N_{AB} \gg N_A N_B$ , which means that accidental triple coincidences of the  $(2 + 1)$  type predominate. Finally, the number of triple coincidences actually measured as such experimentally is therefore written :

$$N_{ABB'} = N_{source}^{(3)} + N_{acc}^{(3)} + N_{acc}^{(2+1)} \approx N_{source}^{(3)} + N_{acc}^{(2+1)}$$

with in the case of an ideal photon source  $N_{\text{source}}^{(3)} = 0$ . This gives a minimum experimental bound to the measurement of the value of  $g^{(2)}(0)$  :

$$g_{GRA}^{(2)}(0) \geq \frac{N_{acc}^{(2+1)} N_A T_{int}}{N_{AB} N_{AB'}}$$

**Q1** Justify that  $g_{GRA}^{(2)}(0) \propto N_{ph}^{inc} \Delta t$  with  $N_{ph}^{inc}$  the number of photon pairs generated by the SPDC source).

**Q2** Based on the previous question, how can  $g_{GRA}^{(2)}(0)$  be minimized? What practical problem(s) might we then encounter?

**Q3** What is the statistical distribution of pair emission events from the non-linear crystal? Deduce the standard deviation  $\sigma(N_i)$  of the uncertainties in counting an average number of events (counts or coincidences)  $N_i$  over the time interval  $T_{int}$ .

**Q4** What average number of events do you need to detect over a time  $T_{int}$  to estimate the rates of counts or coincidences with a relative uncertainty of 1%? With an uncertainty of 10% ? In each case, what is the signal-to-noise ratio (SNR) associated with the measurement?

**Q5** Assuming all detectors and splitters to be perfect, plot the evolution of the value of  $g^{(2)}(0)$  as a function of the number of pairs detected, for measurement times of  $T_{int} = 1s$ , taking a coincidence window  $\Delta t = 10 \text{ ns}$ . What is the minimum value of  $g^{(2)}(0)$  that can be reached for a rate of  $10^5$  pairs per second?

## 2-detector $g(2)(0)$

Experimentation and analysis of results requires accurate estimation of uncertainties.

→ Once the alignment has been optimized, use the knob selector to switch to  $g(2)(0)$  2 det. mode, measuring the unconditioned correlations between the two outputs of the HBT interferometer.

→ Select an integration time window  $T_{int} = 10 \text{ s}$ . Report the corresponding values for  $N_B$ ,  $N_{B'}$  and  $N_{BB'}$ .

**Q6** Calculate the corresponding count and coincidence rates, in number of events per second, and carefully estimate the associated uncertainties.

**Q7** What is the proportion of accidental coincidences between detectors  $A$  and  $B$ ? If necessary, propose a corrected value for the coincidence flux and its uncertainty.

**Q8** Use your previous measurements or the calculation directly returned by the interface to measure a two-detector  $g^{(2)}(0)$  at the Michelson output (now between channels  $A$  and  $B$ ). Carefully estimate the uncertainties in your result.

**Q9** Is the probed SPDC beam a single-photon source?

### Measurement of a 3-detector $g^{(2)}(0)$

↪ Turn off photodiodes.

↪ Carefully swap the coaxial cables of channels  $A$  and  $B$  located behind the photodiodes.

↪ Select an integration time  $T_{\text{int}} = 10 \text{ s}$ .

**Q10** Find the corresponding values for  $N_A$ ,  $N_B$ ,  $N_{B'}$ ,  $N_{AB}$ ,  $N_{AB'}$  and  $N_{ABB'}$ .

**Q11** Find the values of  $g^{(2)}(0)$  at 3 detectors directly calculated via the interface after each integration cycle.

An intrinsic difficulty of the 3-detector set-up is that it relies on the measurement of coincidences between all three channels in the set-up, which, single-photon source or not, is a rare and infrequent event, unless you have sources delivering very high fluxes, which would then threaten to damage the photodiodes.

In other words, for integration times limited to a few seconds, it is almost certain to obtain a number of coincidences  $N_{ABB'} = 0$ . If the calculation then necessarily yields  $g^{(2)}(0) = 0$ , only a reliable estimate of the uncertainties can tell whether the measurement is conclusive or not. This final estimate is then entirely dependent on the uncertainties in the measurements of counts and double coincidence rates on the different detectors.

**Q12** From the measurements of the number of counts and coincidences recorded over the time  $T_{\text{int}}$ , calculate once again the corresponding counts and coincidence rates and their uncertainties.

**Q13** Considering the uncertainties on the counting of the different events independent of each other, give an expression for the relative error  $\Delta g_{\text{GRA}}^{(2)}(0)$  of your measurement of  $g_{\text{GRA}}^{(2)}(0)$ . Which terms dominate the relative error?

**Q14** Assuming that triple coincidences are limited by accidental coincidences  $(2+1)$ , estimate the number of triple coincidences you need to detect to obtain a relative error of 10 pc on the measurement of  $g_{\text{GRA}}^{(2)}(0)$ .

$$N_{AB}(\Delta x) = \frac{N_0}{2} (1 + C \cos(\frac{4\pi \Delta x}{\lambda}))$$

where  $\Delta x$  represents the displacement of the mirror mounted on the piezo-electric stage relative to the optical contact.

↪ Check that the number of coincidences between  $G$  and  $A$  or  $B$  is non-zero, typically of the order of at least a few dozen coincidences per second.

↪ If necessary, improve the set-up: start by blocking one of the Michelson arms, and optimize the coincidences measured on each detector pair by adjusting the position of the corresponding Michelson mirror. Then do the same by blocking the other channel. The number of coincidences provided by each of the two arms should be equivalent.

**Q15** We'll try to obtain the interference period, to check that it corresponds to the wavelength of the photons converted by SPDC. Applying the Shannon-Nyquist theorem, what is the maximum carriage distance between two consecutive points that must be respected in order to correctly determine the wavelength? Check that the minimum shifting distance of the wedge allows this criterion to be met.

↪ Open the Kinesis software for controlling the piezoelectric stage. Use the “**Closed loop**” mode to position the stage.

↪ Use the software's “**step-by-step**” translation mode to move the stage by its minimum displacement increment.

**Q16** For a well-chosen counting time  $T_{\text{int}}$ , plot the evolution of the number of coincidences  $N_{AB}$  as a function of the displacement of the piezoelectric wedge. Add error bars to represent your uncertainties about the coincidence count, explaining how you evaluate the uncertainties. Also take into account the uncertainty in the position of the shim at each translation step.

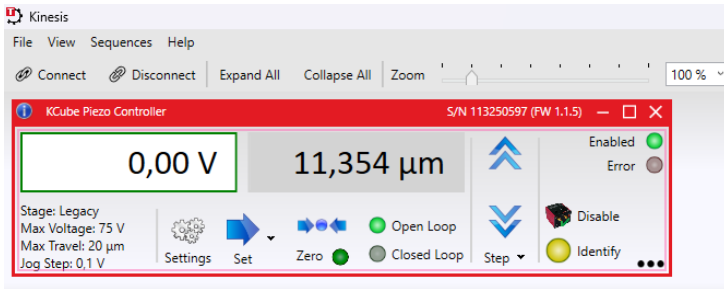


Figure 6.6: Fenêtre de contrôle du logiciel Kinesis.

**Q17** Using a weighted least-squares method, fit a sinusoidal model to your experimental data. What is the period of this sinusoid? Can you find the wavelength of the photon injected into the Michelson interferometer? Evaluate the contrast of these interference fringes.

## 2.4 Conclusion and interpretations

The two main results of this set-up seem strongly contradictory:

- The measurement of  $g^{(2)}(0)$  by HBT interferometry suggests that a photon is never split in two by a splitter, but rather takes one or other of the output paths.
- However, the observation of interference bands at the Michelson's output can only be explained by a wave model.

This paradox is that of **wave-corpuscule duality**. This notion has a subtle link with the notion of quantum indiscernibility, and in our experiment, with the concept of **which-path information**.

**Q18** At the output of a Michelson, what can you say about the path information of each photon detected?

**Q19** If the Michelson's two mirrors were now replaced by two detectors, what would we observe in terms of counts and (conditioned) coincidences on them? What can you say this time about the path taken by the photons successively detected?

**Q20** Using equation (6.3), justify that the observation of interference is the consequence of the indistinguishability between several quantum paths. What parallels can you draw with the HOM experiment?

**Q21** In your own words, explain what you think is fundamentally wave-like and what is fundamentally corpuscular in the description of a single photon.

## Appendix : Quantum states of light

### Coherent states

In addition to the single photon state, there are other quantum states of importance in describing the different states of light.

Light can be placed in a so-called coherent “state”, which is a pure quantum state written as a superposition:

$$|\Psi_{\text{coh.}}\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{+\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

By construction, the coherent state is one whose behavior reproduces that of coherent light described in classical physics: it is therefore typically the state associated with laser light. The probability of measuring  $n$  photons is then :

$$P_{\text{laser}}(n) = |\langle n | \Psi_{\text{coh.}} \rangle|^2 = e^{-|\alpha|^2} \frac{\alpha^{2n}}{n!}$$

The statistical distribution of the number of photons in the mode of a coherent source is a **Poisson law**. It is not a single-photon source.

We can look at the  $g^{(2)}(0)$  of such a source, particularly in the case of a strongly attenuated source ( $\alpha \rightarrow 0$ ). We then see that the source converges to the empty state of the  $|0\rangle$  field, but its statistical properties persist.

At low flux, we can intuitively understand that the coincidence events measured in an HBT interferometer are almost exclusively related to the weight of the two-photon  $|2\rangle$  component of the field: when two photons arrive at a splitter, 50% of the time they emerge separated. The probability of coincidence  $P_{AB}$  is then typically equal to  $P(2)/2$ . The  $g^{(2)}(0)$  of the source can therefore be quickly estimated by the ratio  $P(2)/P(1)^2$ , which expresses the relative frequency of coincidence events compared to independent detection events on the two channels. For a coherent source,

$$g^{(2)}(0) \approx 2P(2)/P(1)^2 = 2 \frac{\alpha^4}{2\alpha^2 \cdot \alpha^2} = 1$$

This ratio does not depend on the intensity of the source: even if the source is strongly attenuated (and the average number of photons in the mode lowered), the relative weight of the 2-photon states compared to the 1-photon states remains the same. The  $g^{(2)}(0)$  is always greater than 1.

### (Pseudo-) Thermal state

When the field of a mode results from a statistical mixture of several incoherent, statistically independent contributions, the measured quantum state is no longer a pure state. It can only be written using more sophisticated tools, such as the density matrix. This is the case with so-called **thermal**, or **chaotic** sources: the field of a mode is then the resultant of numerous independent spontaneous emission processes. The state is then written :

$$\rho_{\text{th}} = \sum_n \frac{\langle \hat{n} \rangle^n}{(1 + \langle \hat{n} \rangle)^{n+1}} |n\rangle \langle n|$$

The density matrix is diagonal, which corresponds to an incoherent statistical mixture of Fock states. The probability of measuring  $n$  photons is given by :

$$P_{\text{th}}(n) = \frac{\langle \hat{n} \rangle^n}{(1 + \langle \hat{n} \rangle)^{n+1}}$$

This factor results from the laws of statistical physics applied to a mode of the electromagnetic field considered in thermodynamic equilibrium at temperature  $T$ . The average number  $\langle \hat{n} \rangle$  of photons in the mode is then given by the Bose-Einstein statistic:

$$\langle \hat{n} \rangle = \frac{1}{e^{\beta \hbar \omega} - 1}$$

Furthermore, the geometric progression in  $n$  of the  $P(n)$  terms is directly related to the incoherent nature of the source. For independent spontaneous emission events associated with a probability  $\lambda^2$  of emission, then the probability of obtaining a number  $n$  of photons is proportional to  $\lambda^{2n}$  :

$$P_{\text{th}}(n) = k \lambda^{2n}$$

with the closure relation

$$\sum_n P_{\text{th}}(n) = 1 = k \sum_n \lambda^{2n} = \frac{k}{1 - \lambda^2}$$

which shows that  $k = 1 - \lambda^2$ . We can then easily identify :

$$P_{\text{th}}(n) = (1 - \lambda^2) \lambda^{2n} = \left( \frac{1}{1 + \langle \hat{n} \rangle} \right) \left( \frac{\langle \hat{n} \rangle}{1 + \langle \hat{n} \rangle} \right)^n \longrightarrow \lambda^2 = \frac{\langle \hat{n} \rangle}{1 + \langle \hat{n} \rangle} = e^{-\beta \hbar \omega}$$

So the thermal state can be written as  $\rho_{\text{th}} = (1 - \lambda^2) \sum_n \lambda^{2n} |n\rangle \langle n|$ . Decreasing the intensity of the source decreases the average number of photons, i.e. we are in the limit  $\lambda \rightarrow 0$ . In this limit, the density matrix converges to that of the pure  $|0\rangle_i$  state: we gradually obtain the field vacuum.

Furthermore, we have  $\lambda^2 \approx \langle \hat{n} \rangle$ , so that  $P_{th}(n) \approx (\langle \hat{n} \rangle)^n$ . As in the previous case, we can quickly estimate a value for  $g^{(2)}(0)$  :

$$g^{(2)}(0) \approx 2P(2)/P(1)^2 = 2 \frac{\langle \hat{n} \rangle^2}{\langle \hat{n} \rangle \langle \hat{n} \rangle} = 2$$

Here again, the ratio does not depend on the intensity of the source: even with an extremely attenuated source, the expected number of coincidences relative to the average flux received is a non-zero constant in the case of the thermal source.

### Quantum state generated by SPDC

In second-order nonlinear optics processes such as SPDC, a photon from the pump beam is converted into a pair of photons *signal* and *idler* (complementary). The reciprocal process also exists: second harmonic generation.

In the quantum-mechanical sense, SPDC is a coherent process: the state of the field is obtained by a unitary evolution from a pure initial state to a final state that is also pure. In the vocabulary of optics, non-linear processes like SPDC are coherent because they are generated at each instant by a coherent field, namely the laser field driving the laser-matter interaction.

The initial state is the vacuum of the electromagnetic field in the two output modes,  $|0_s\rangle|0_i\rangle$ . The Hamiltonian of the SPDC process that applies to this state is written :

$$\hat{H}_{\text{SPDC}} \propto \chi^{(2)} \left( \hat{a}_s^\dagger \hat{a}_i^\dagger - \hat{a}_s \hat{a}_i \right)$$

and shows the creation or annihilation of pairs. The associated unitary evolution is written :

$$|\psi(t)\rangle_{\text{SPDC}} = e^{-i\hat{H}_{\text{SPDC}}t/\hbar} |0_s\rangle|0_i\rangle$$

and it can be shown, with the help of several theorems describing the normal ordering of operators, that the field state generated by SPDC is written as <sup>3</sup>:

$$|psi(r)\rangle_{\text{SPDC}} = \text{sech}(r) \sum_{n=0}^{+\infty} \tanh^n(r) |n\rangle_s |n\rangle_i$$

where  $r$  is a complex number, called the squeezing parameter, and describes the interaction with the pump – its value depends on the complex amplitude

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<sup>3</sup>(see, for example, Stephen M. Barnett, Paul M. Radmore, *Methods in theoretical quantum optics*, Oxford series in optical and imaging sciences 15, page 76, eq 3.7.50, (1997) ; or D. B. Horoshko et al, *Thermal-difference states of light: Quantum states of heralded photons*, Phys. Rev. A **100**, 053831(2019)

of the pump field, the nonlinear response coefficient  $\chi^{(2)}$ , etc.

The state presents perfect photon number correlations between the mode of the *signal* beam and that of the *idler* beam: if we measure a number  $m$  of photons in the  $s$  mode, then the  $i$  mode contains exactly the same number  $m$  of photons. It is therefore a highly entangled state.

The quantum state associated with only one of the two modes (e.g. the *signal*) is calculated by performing the partial trace on the states of the other mode (i.e. the *idler*). The density matrix of the total system is written :

$$\rho = |\psi(r)\rangle\langle\psi(r)| = \text{sech}^2(r) \sum_{n,m} \tanh^{n+m}(r) |n\rangle_s \langle m|_s \otimes |n\rangle_i \langle m|_i$$

The partial trace of the idler mode is :

$$\begin{aligned} \rho_s = \text{Tr}_i(\rho) &= \sum_k \langle k|_i \rho |k\rangle_i \\ &= \text{sech}^2(r) \sum_{k,n,m} \tanh^{n+m}(r) \delta_{k,n} \delta_{k,m} |n\rangle_s \langle m|_s \Rightarrow k = n = m \\ &= \text{sech}^2(r) \sum_n \tanh^{2n}(r) |n\rangle_s \langle n|_s \\ &= (1 - \lambda^2) \sum_n \lambda^{2n} |n\rangle_s \langle n|_s \end{aligned}$$

We find that the quantum state associated with one (of the two) SPDC modes is in a **thermal state** as described above: a statistical mixture of Fock states, weighted by geometrically progressing probabilities. We directly identify  $\lambda^2 = \tanh^2(r) = \frac{\langle \hat{n} \rangle}{1 + \langle \hat{n} \rangle} = e^{-\beta \hbar \omega}$ .

To sum up: the *two mode* quantum state resulting from SPDC is indeed a pure state, a **coherent superposition of two-photon states**. The quantum correlations it possesses are measurable via joint measurements on the two photons exploiting quantum interference effects, like measurements of the Bell parameter.

Conversely, detection measurements performed on only one of the two modes ignore, by definition, the correlations between the two modes. Phase information - initially derived from the phase of the pump beam - is therefore lost. The result of each measurement on an individual mode appears to be dictated by independent spontaneous emission events, with probability proportional to  $\lambda^2$  : **the single-mode quantum state is a statistical mixture of number states according to a thermal distribution.**

Furthermore, for a low-intensity SPDC source, the field state converges to the empty state:  $\lambda^2 \rightarrow 0 \Rightarrow \rho_s \rightarrow |0\rangle_s \langle 0|_s$ .

### Conditional detection of a heralding photon

Although SPDC is a photon pair generation process, the statistical distribution of photons in one of the two generated modes does not behave at all like a single photon source. The aim is to place one of the two beams in a state as close as possible to the Fock  $|1\rangle$  state. This means actively exploiting the correlation between the two beams.

The strategy is as follows: we know that if the first mode contains exactly one photon, then the second mode contains exactly one photon too. We can therefore perform a measurement on the first mode. The detection of a photon on this first channel is the heralding signal: this event indicates that the twin beam also contains a photon at the same instant. The heralding photon is of course no longer usable, as it has been absorbed. But the heralded photon is still available. In a very short time window, synchronized with the announcement photon, the state of the heralded field is known: it is a state with "at least one photon".

For detectors with unit efficiency, but unable to resolve the photon number, we can define two measurement operator operator  $\Pi_{\text{off},i} = |0\rangle_i \langle 0|_i$  referring to no detection on mode  $i$ , detection operator  $\Pi_{\text{on},i} = \mathbb{1}_i - |0\rangle_i \langle 0|_i$  corresponding to the detection of at least one photon on mode  $i$ .

We can now turn our attention to the quantum state of the  $s$  mode, measured in conjunction with the observation of a photon on the  $s$  channel:

$$\begin{aligned} \rho_s^{\text{cond.}} &= \text{Tr}_i(\Pi_{\text{on},i} |\psi(r)\rangle \langle \psi(r)|) = \sum_k \langle k|_i \rho |k\rangle_i = (1 - \lambda^2) \sum_{n=1}^{+\infty} \lambda^{2n} |n\rangle_s \langle n|_s \\ &= \rho_s - (1 - \lambda^2) |0\rangle_s \langle 0|_s \end{aligned}$$

More properly renormalized (the trace must be equal to 1), the matrix is written:

$$\rho_s^{\text{cond.}} = \frac{1 - \lambda^2}{\lambda^2} \left( \frac{1 - \lambda^2}{-} |0\rangle_s \langle 0|_s \right)$$

The measured conditional state is the previously calculated thermal state, truncated by the vacuum contribution of the  $|0\rangle_s \langle 0|_s$  field. When the field is sufficiently weak, the state obtained is the pure one-photon state:

$$\lambda^2 \rightarrow 1 \Rightarrow \rho_s^{\text{cond.}} \rightarrow |1\rangle_s \langle 1|_s$$